Chapter 6

Some Continuous Probability Distributions Probability & Statistics for Engineers & Scientists

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Continuous Uniform Distribution Probability & Statistics for Engineers & Scientists

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Figure 6.1 The density function for a random variable on the interval [1,3]



Theorem 6.1



The mean and variance of the uniform distribution are $\mu = \frac{A+B}{2} \text{ and } \sigma^2 = \frac{(B-A)^2}{12}.$

Normal Distribution

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Figure 6.2 The normal curve



Figure 6.3 Normal curves with $\mu_1 < \mu_2$ and $\sigma_1 = \mu_2$



Figure 6.4 Normal curves with $\mu_1 = \mu_2$ and $\sigma_1 < \sigma_2$



Figure 6.5 Normal curves with $\mu_1 < \mu_2$ and $\sigma_1 < \mu_2$



Theorem 6.2



The mean and variance of $n(x; \mu, \sigma)$ are μ and σ^2 , respectively. Hence, the standard deviation is σ .

Areas under the Normal Curve

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Figure 6.6 $P(x_1 < X < x_2) = \text{area of}$ the shaded region



Figure 6.7 $P(x_1 < X < x_2)$ for different normal curves



Definition 6.1



The distribution of a normal random variable with mean 0 and variance 1 is called a **standard normal distribution**.

Figure 6.8 The original and transformed normal distributions





Figure 6.9 Areas for Example 6.2



Figure 6.10 Areas for Example 6.3





Figure 6.11 Area for Example 6.4





Figure 6.12 Area for Example 6.5



Figure 6.13 Areas for Example 6.6



Applications of the Normal Distribution Probability & Statistics for Engineers & Scientists

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Figure 6.14 Area for Example 6.7





Figure 6.15 Area for Example 6.8





Figure 6.16 Area for Example 6.9



Figure 6.17 Specifications for Example 6.10



Figure 6.18 Area for Example 6.11





Figure 6.19 Area for Example 6.12





Figure 6.20 Area for Example 6.13





Figure 6.21 Area for Example 6.14



Normal Approximation to the Binomial Probability & Statistics for Engineers & Scientists



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Theorem 6.3



If X is a binomial random variable with mean $\mu = np$ and variance $\sigma^2 = npq$, then the limiting form of the distribution of

$$Z = \frac{X - np}{\sqrt{npq}},$$

as $n \to \infty$, is the standard normal distribution n(z; 0, 1).

Figure 6.22 Normal approximation of *b*(*x*; 15,0.4)



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Figure 6.23 Normal approximation of b(x; 15, 0.4) and $\sum_{x=7}^{9} b(x; 15, 0.4)$



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Figure 6.24 Histogram for *b*(*x*; 6, 0.2)





Figure 6.25 Histogram for *b*(*x*; 15, 0.2)





Table 6.1 Normal Approximation andTrue Cumulative Binomial Probabilities

| | p = 0.05, n = 10 | | p = 0.10, n = 10 | | p = 0.50, n = 10 | |
|--|--|---|--|--|--|---|
| r | Binomial | Normal | Binomial | Normal | Binomial | Normal |
| 0 | 0.5987 | 0.5000 | 0.3487 | 0.2981 | 0.0010 | 0.0022 |
| 1 | 0.9139 | 0.9265 | 0.7361 | 0.7019 | 0.0107 | 0.0136 |
| 2 | 0.9885 | 0.9981 | 0.9298 | 0.9429 | 0.0547 | 0.0571 |
| 3 | 0.9990 | 1.0000 | 0.9872 | 0.9959 | 0.1719 | 0.1711 |
| 4 | 1.0000 | 1.0000 | 0.9984 | 0.9999 | 0.3770 | 0.3745 |
| 5 | | | 1.0000 | 1.0000 | 0.6230 | 0.6255 |
| 6 | | | | | 0.8281 | 0.8289 |
| 7 | | | | | 0.9453 | 0.9429 |
| 8 | | | | | 0.9893 | 0.9864 |
| 9 | | | | | 0.9990 | 0.9978 |
| 10 | | | | | 1.0000 | 0.9997 |
| | p = 0.05 | | | | | |
| | | | p = | 0.05 | | |
| | n = | = 20 | $\frac{p-1}{n}$ | = 50 | n = | 100 |
| r | n = Binomial | = 20 Normal | $\frac{p-n}{n} = $ Binomial | = 50 Normal | n =Binomial | 100 Normal |
| r 0 | n = Binomial 0.3585 | = 20 Normal 0.3015 | $\frac{p-n}{n} = \frac{1}{0.0769}$ | = 50 Normal 0.0968 | n = Binomial 0.0059 | 100 Normal 0.0197 |
| r 0 1 | n = Binomial 0.3585 0.7358 | = 20 Normal 0.3015 0.6985 | | = 50 Normal 0.0968 0.2578 | n = Binomial 0.0059 0.0371 | 100 Normal 0.0197 0.0537 |
| r 0 1 2 | n = Binomial 0.3585 0.7358 0.9245 | = 20 Normal 0.3015 0.6985 0.9382 | $ \frac{p - 1}{n = 1} $ Binomial 0.0769 0.2794 0.5405 | | n = Binomial 0.0059 0.0371 0.1183 | 100 Normal 0.0197 0.0537 0.1251 |
| r 0 1 2 3 | n = Binomial 0.3585 0.7358 0.9245 0.9841 | = 20 Normal 0.3015 0.6985 0.9382 0.9948 | | | n = 0.0059 0.0371 0.1183 0.2578 | 100 Normal 0.0197 0.0537 0.1251 0.2451 |
| r 0 1 2 3 4 | n = Binomial 0.3585 0.7358 0.9245 0.9841 0.9974 | = 20 Normal 0.3015 0.6985 0.9382 0.9948 0.9998 | $\begin{array}{r} p-\\ n=\\ \hline \textbf{Binomial}\\ 0.0769\\ 0.2794\\ 0.5405\\ 0.7604\\ 0.8964 \end{array}$ | $\begin{array}{r} \textbf{0.03} \\ \hline \textbf{Normal} \\ 0.0968 \\ 0.2578 \\ 0.5000 \\ 0.7422 \\ 0.9032 \end{array}$ | n = 0.0059 0.0371 0.1183 0.2578 0.4360 | 100 Normal 0.0197 0.0537 0.1251 0.2451 0.4090 |
| r 0 1 2 3 4 5 | n = Binomial 0.3585 0.7358 0.9245 0.9841 0.9974 0.9997 | = 20 Normal 0.3015 0.6985 0.9382 0.9948 0.9998 1.0000 | $\begin{array}{r} p-\\ \hline n=\\ \hline 0.0769\\ 0.2794\\ 0.5405\\ 0.7604\\ 0.8964\\ 0.9622 \end{array}$ | $\begin{array}{r} \textbf{0.03} \\ \hline \textbf{Normal} \\ 0.0968 \\ 0.2578 \\ 0.5000 \\ 0.7422 \\ 0.9032 \\ 0.9744 \end{array}$ | n = 0.0059 0.0371 0.1183 0.2578 0.4360 0.6160 | 100 Normal 0.0197 0.0537 0.1251 0.2451 0.4090 0.5910 |
| r 0 1 2 3 4 5 6 | n = Binomial 0.3585 0.7358 0.9245 0.9841 0.9974 0.9997 1.0000 | = 20 Normal 0.3015 0.6985 0.9382 0.9948 0.9998 1.0000 1.0000 | $p = \frac{p}{n} = \frac{n}{0.0769}$ 0.0769 0.2794 0.5405 0.7604 0.8964 0.9622 0.9882 | $\begin{array}{r} \hline 0.03 \\ \hline \hline 0.0968 \\ 0.2578 \\ 0.5000 \\ 0.7422 \\ 0.9032 \\ 0.9744 \\ 0.9953 \\ \end{array}$ | n = Binomial 0.0059 0.0371 0.1183 0.2578 0.4360 0.6160 0.7660 | 100 Normal 0.0197 0.0537 0.1251 0.2451 0.4090 0.5910 0.7549 |
| $egin{array}{c} m{r} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}$ | n = Binomial 0.3585 0.7358 0.9245 0.9841 0.9974 0.9997 1.0000 | = 20 Normal 0.3015 0.6985 0.9382 0.9948 0.9998 1.0000 1.0000 | $p = \frac{p}{n} = \frac{n}{100000000000000000000000000000000000$ | $\begin{array}{r} \textbf{0.03} \\ \hline \textbf{Normal} \\ 0.0968 \\ 0.2578 \\ 0.5000 \\ 0.7422 \\ 0.9032 \\ 0.9744 \\ 0.9953 \\ 0.9994 \end{array}$ | $\begin{array}{r} n = \\ \hline \textbf{Binomial} \\ 0.0059 \\ 0.0371 \\ 0.1183 \\ 0.2578 \\ 0.4360 \\ 0.6160 \\ 0.7660 \\ 0.8720 \end{array}$ | $\begin{array}{r} 100\\ \hline \textbf{Normal}\\ 0.0197\\ 0.0537\\ 0.1251\\ 0.2451\\ 0.4090\\ 0.5910\\ 0.7549\\ 0.8749 \end{array}$ |
| r 0 1 2 3 4 5 6 7 8 | n = Binomial 0.3585 0.7358 0.9245 0.9841 0.9974 0.9997 1.0000 | = 20 Normal 0.3015 0.6985 0.9382 0.9948 0.9998 1.0000 1.0000 | $\begin{array}{r} p-\\ \hline n=\\ \hline n=\\ 0.0769\\ 0.2794\\ 0.5405\\ 0.7604\\ 0.8964\\ 0.9622\\ 0.9882\\ 0.9968\\ 0.9992\\ \end{array}$ | $\begin{array}{r} 0.03 \\ \hline \hline 0.0968 \\ 0.2578 \\ 0.5000 \\ 0.7422 \\ 0.9032 \\ 0.9744 \\ 0.9953 \\ 0.9994 \\ 0.9999 \end{array}$ | $\begin{array}{l} n = \\ \hline \textbf{Binomial} \\ 0.0059 \\ 0.0371 \\ 0.1183 \\ 0.2578 \\ 0.4360 \\ 0.6160 \\ 0.7660 \\ 0.8720 \\ 0.9369 \end{array}$ | $\begin{array}{r} 100\\ \hline \textbf{Normal}\\ 0.0197\\ 0.0537\\ 0.1251\\ 0.2451\\ 0.4090\\ 0.5910\\ 0.7549\\ 0.8749\\ 0.9463\\ \end{array}$ |
| $\begin{array}{c} r \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{array}$ | n = Binomial 0.3585 0.7358 0.9245 0.9841 0.9974 0.9997 1.0000 | = 20 Normal 0.3015 0.6985 0.9382 0.9948 0.9998 1.0000 1.0000 | $\begin{array}{r} p-\\ \hline n=\\ \hline n=\\ 0.0769\\ 0.2794\\ 0.5405\\ 0.7604\\ 0.8964\\ 0.9622\\ 0.9882\\ 0.9968\\ 0.9992\\ 0.9998\\ \hline \end{array}$ | $\begin{array}{r} \textbf{0.03} \\ \hline \textbf{Normal} \\ 0.0968 \\ 0.2578 \\ 0.5000 \\ 0.7422 \\ 0.9032 \\ 0.9744 \\ 0.9953 \\ 0.9994 \\ 0.9999 \\ 1.0000 \end{array}$ | $\begin{array}{r} n = \\ \hline \textbf{Binomial} \\ 0.0059 \\ 0.0371 \\ 0.1183 \\ 0.2578 \\ 0.4360 \\ 0.6160 \\ 0.7660 \\ 0.8720 \\ 0.9369 \\ 0.9718 \end{array}$ | $\begin{array}{r} 100\\ \hline \textbf{Normal}\\ 0.0197\\ 0.0537\\ 0.1251\\ 0.2451\\ 0.4090\\ 0.5910\\ 0.7549\\ 0.8749\\ 0.9463\\ 0.9803\\ \end{array}$ |

Figure 6.26 Area for Example 6.15



Figure 6.27 Area for Example 6.15



Gamma and Exponential Distributions

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Definition 6.2



The gamma function is defined by

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx, \quad \text{for } \alpha > 0.$$

Figure 6.28 Gamma distributions



Theorem 6.4



The mean and variance of the gamma distribution are

$$\mu = \alpha \beta$$
 and $\sigma^2 = \alpha \beta^2$.

Corollary 6.1



The mean and variance of the exponential distribution are

 $\mu = \beta$ and $\sigma^2 = \beta^2$.

Chi-Squared Distributions

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Theorem 6.5



The mean and variance of the chi-squared distribution are

 $\mu = v$ and $\sigma^2 = 2v$.

Beta Distribution

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Definition 6.3



A **beta function** is defined by

$$B(\alpha,\beta) = \int_0^1 x^{\alpha-1} (1-x)^{\beta-1} dx = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, \text{ for } \alpha,\beta > 0,$$

where $\Gamma(\alpha)$ is the gamma function.

Theorem 6.6



The mean and variance of a beta distribution with parameters α and β are

$$\mu = \frac{\alpha}{\alpha + \beta}$$
 and $\sigma^2 = \frac{\alpha \beta}{(\alpha + \beta)^2 (\alpha + \beta + 1)}$,

respectively.

Lognormal Distribution

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Figure 6.29 Lognormal distributions





Theorem 6.7



The mean and variance of the lognormal distribution are

$$\mu = e^{\mu + \sigma^2/2}$$
 and $\sigma^2 = e^{2\mu + \sigma^2} (e^{\sigma^2} - 1)$.

Weibull Distribution (Optional) Probability & Statistics for Engineers & Scientists

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Theorem 6.8



The mean and variance of the Weibull distribution are

$$\mu = \alpha^{-1/\beta} \Gamma\left(1 + \frac{1}{\beta}\right) \text{ and } \sigma^2 = \alpha^{-2/\beta} \left\{ \Gamma\left(1 + \frac{2}{\beta}\right) - \left[\Gamma\left(1 + \frac{1}{\beta}\right)\right]^2 \right\}$$

Figure 6.30 Weibull distributions $(\alpha = 1)$



Potential **Misconceptions** and Hazards; **Relationship to** Material in Other Chapters

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