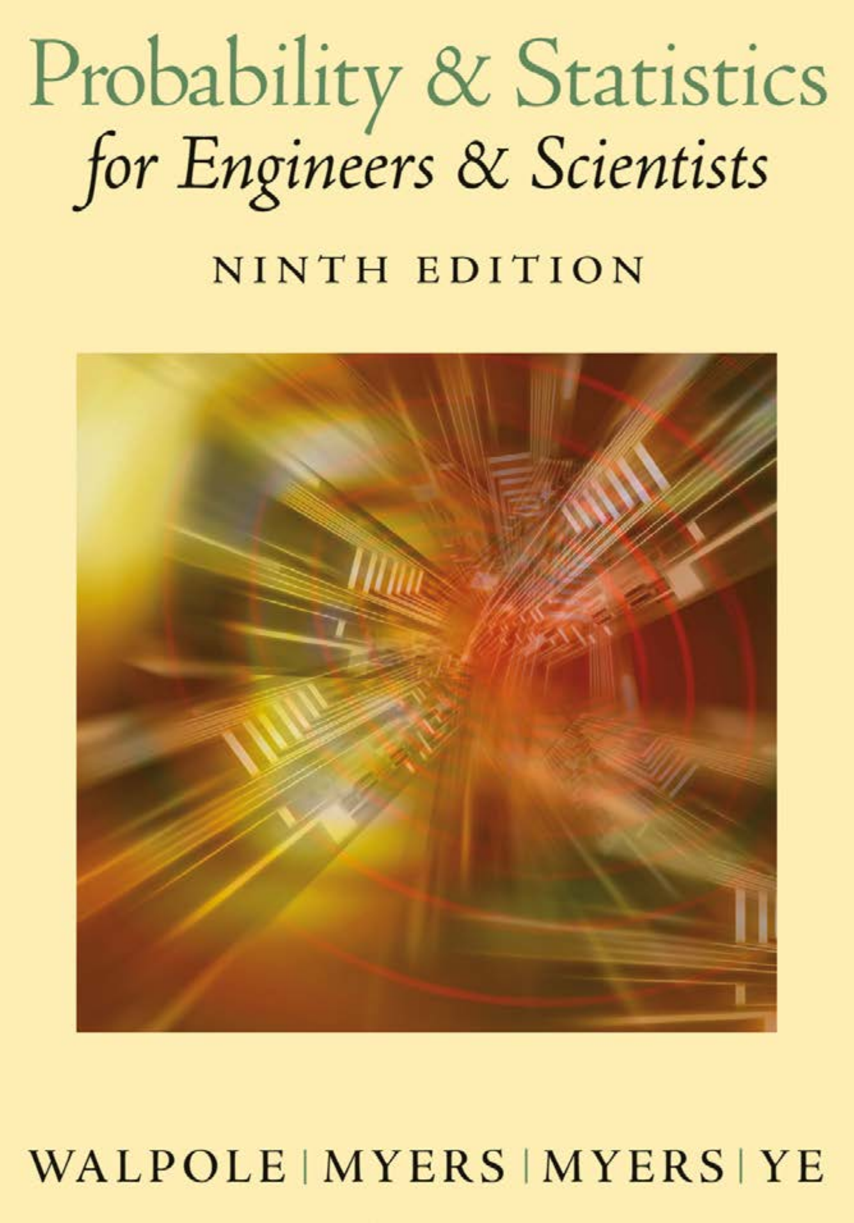
# Chapter 6

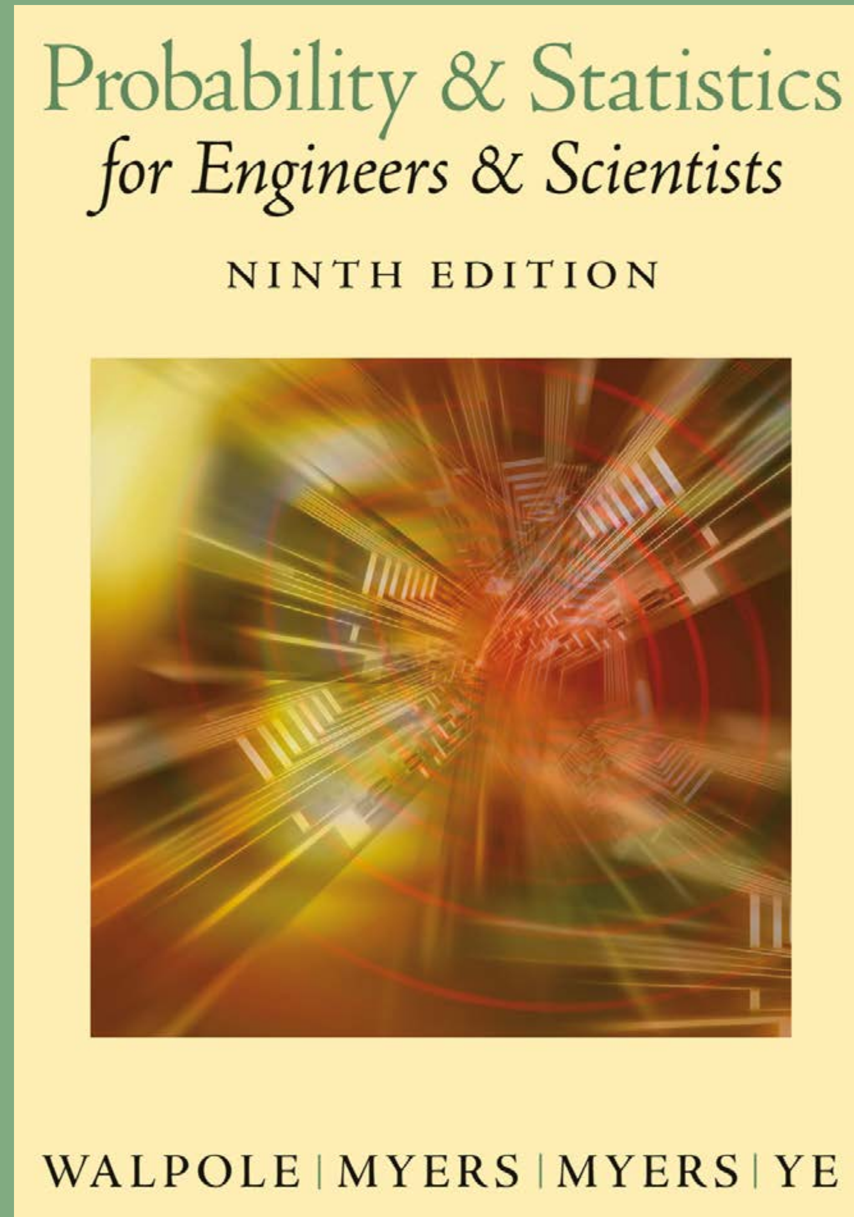
## Some Continuous Probability Distributions



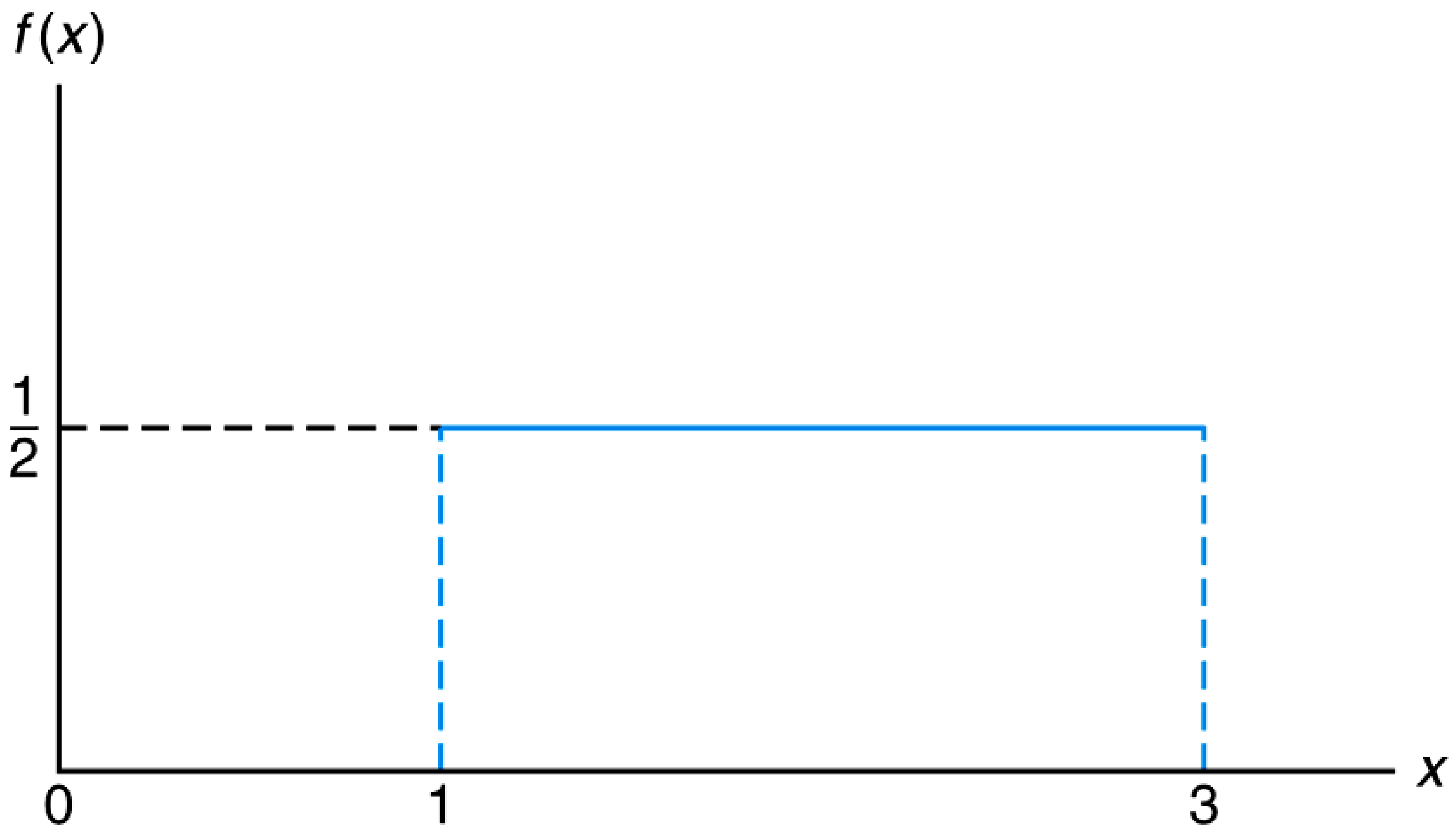

PEARSON Addison Wesley

# Section 6.1

Continuous Uniform Distribution



# **Figure 6.1** The density function for a random variable on the interval $[1,3]$



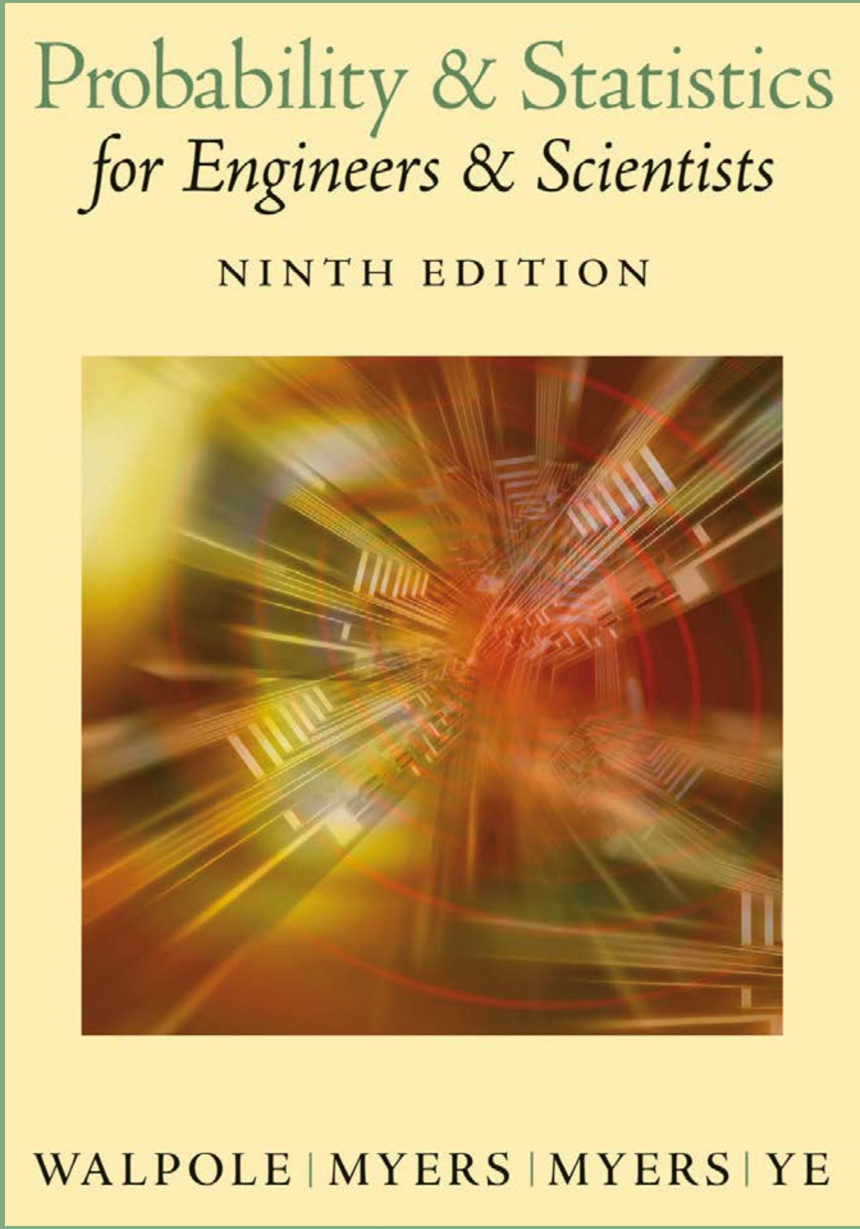
# Theorem 6.1

The mean and variance of the uniform distribution are

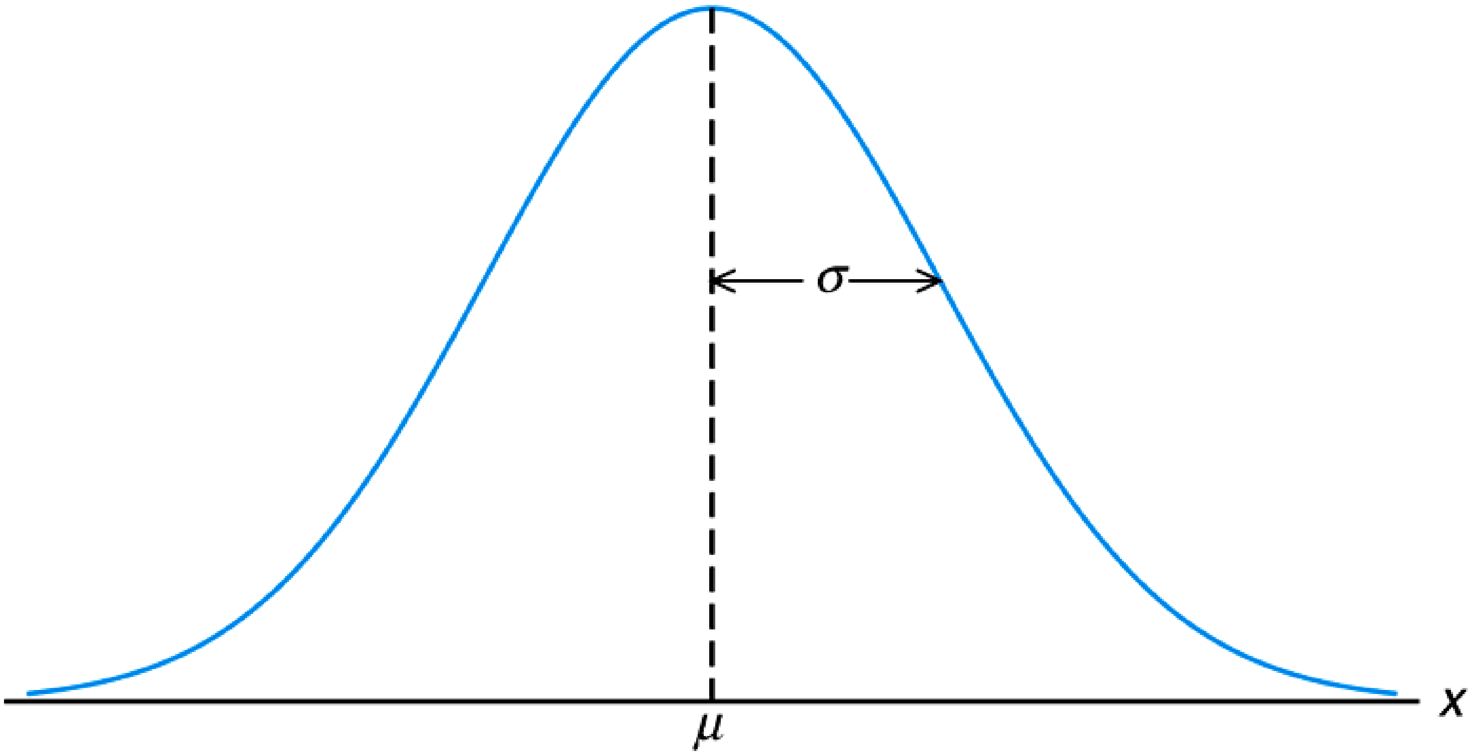
$$\mu = \frac{A+B}{2} \text{ and } \sigma^2 = \frac{(B-A)^2}{12}.$$

# Section 6.2

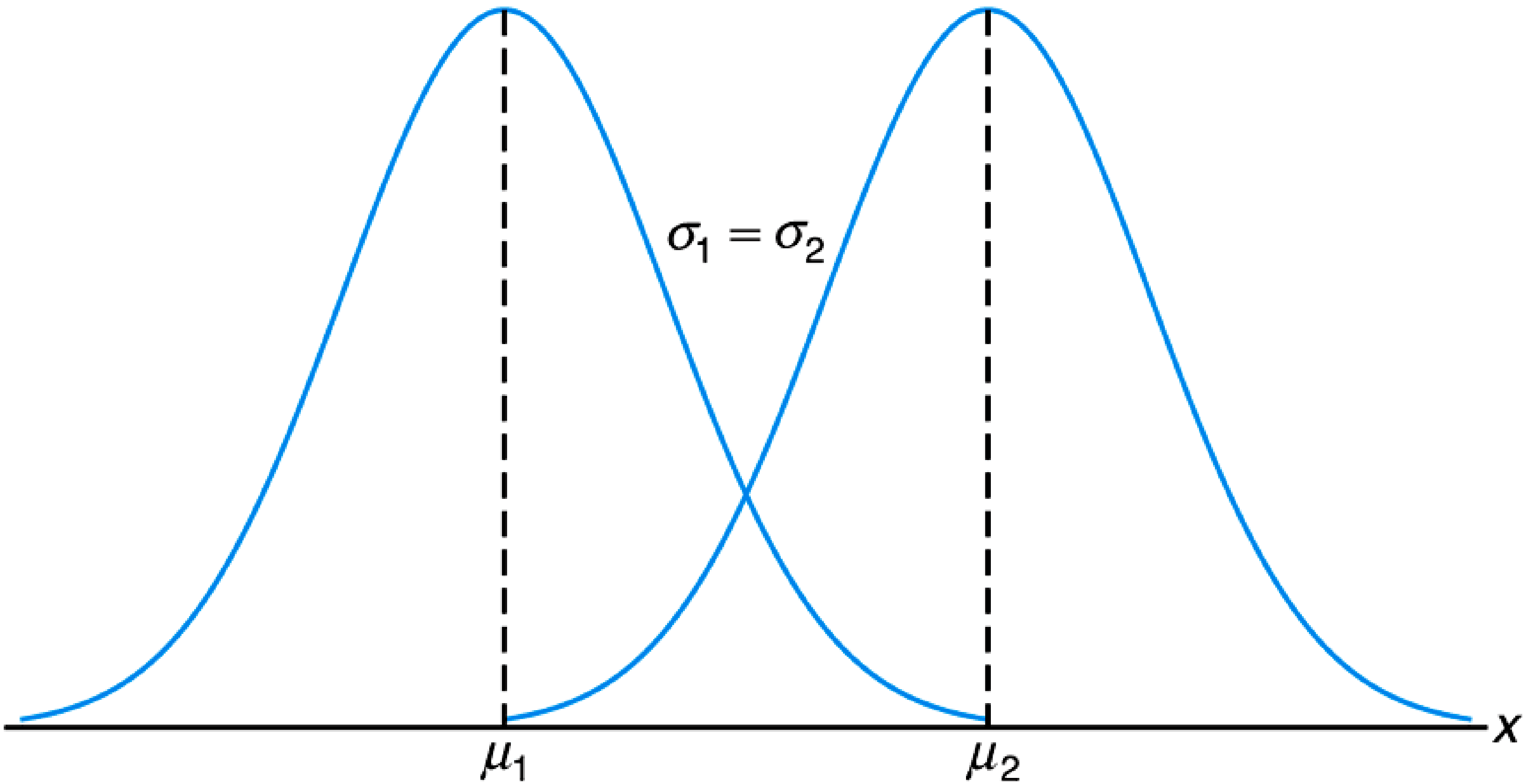
Normal Distribution



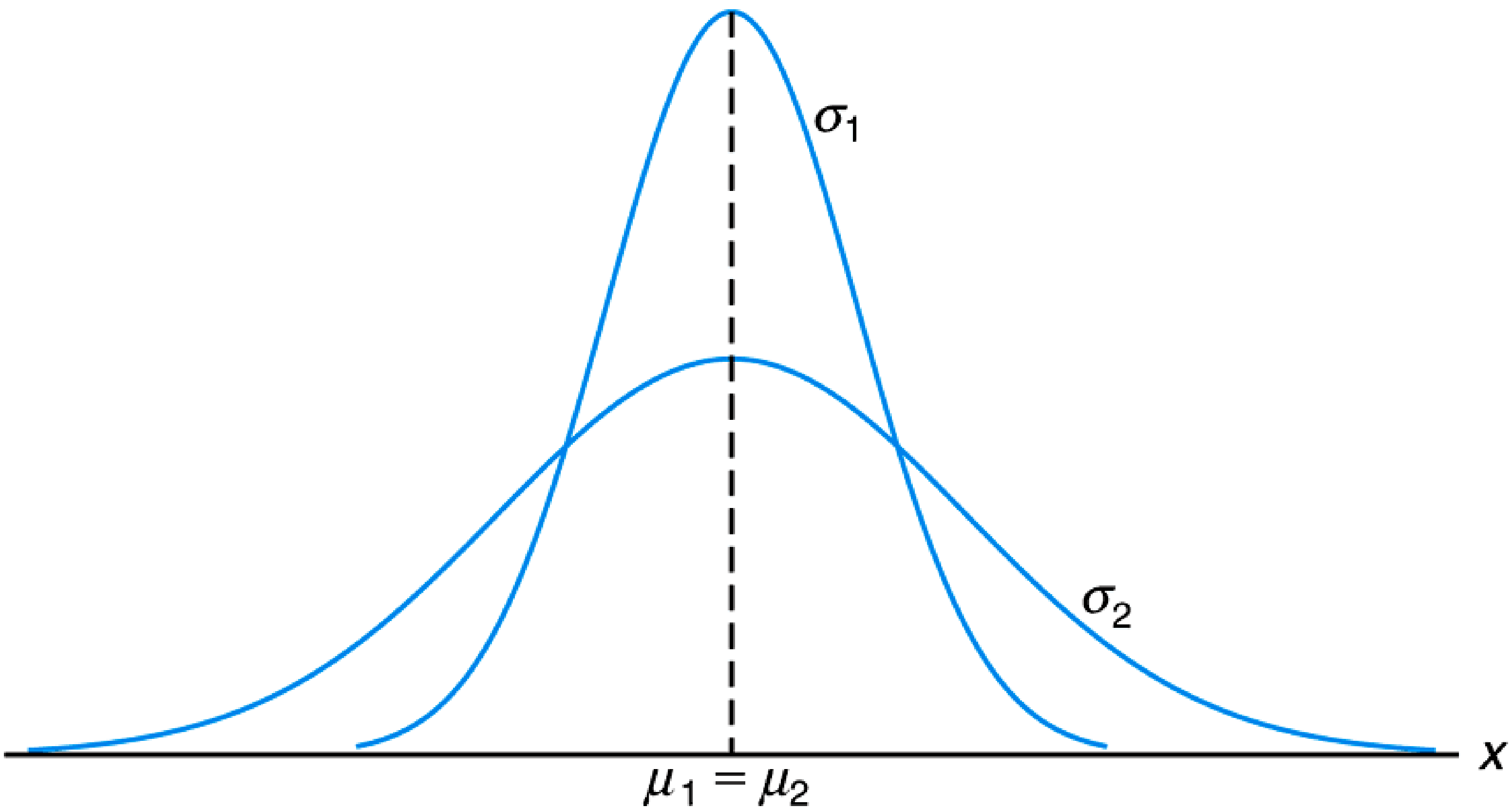
# **Figure 6.2** The normal curve



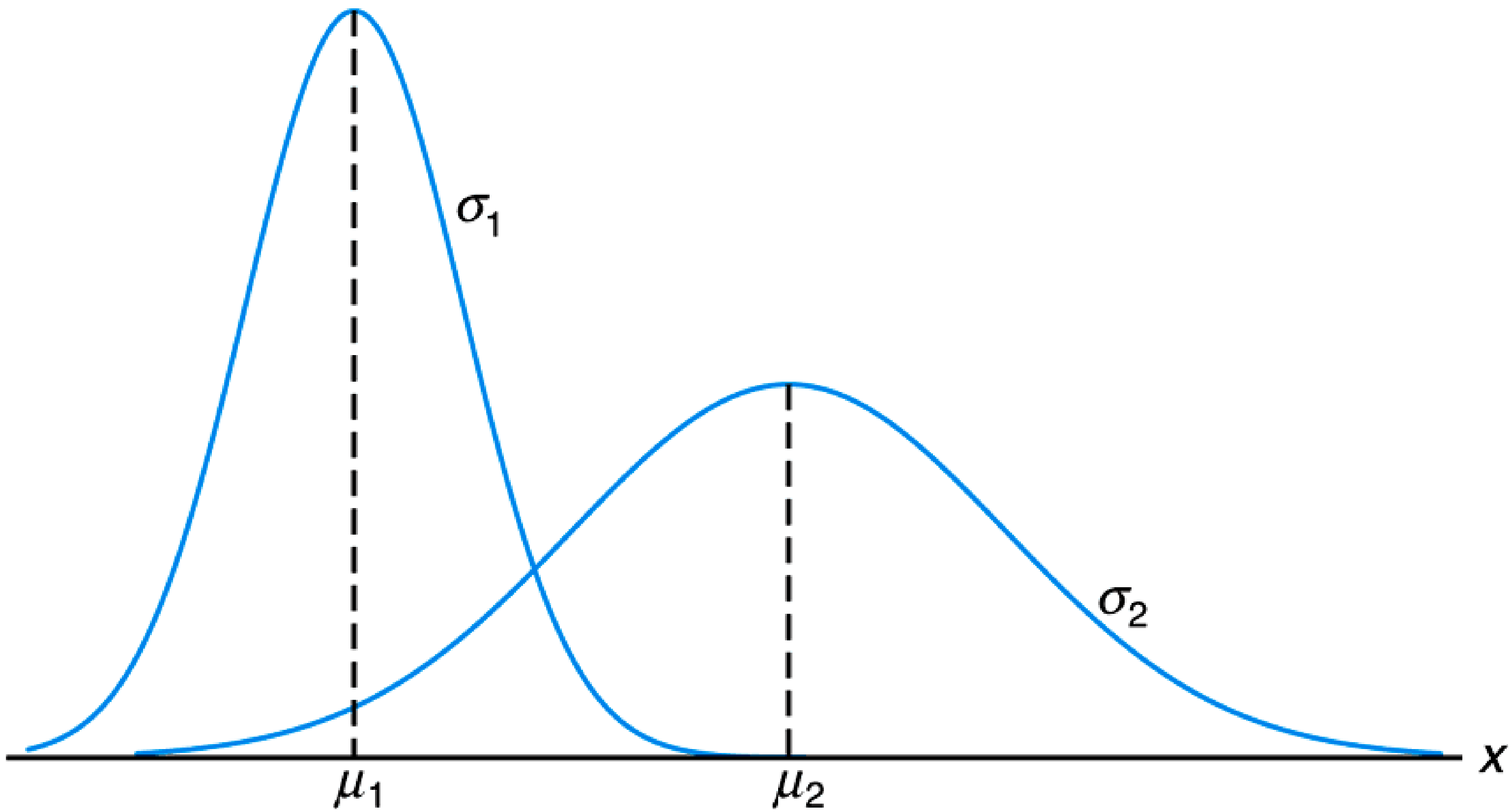
# **Figure 6.3** Normal curves with $\mu_1 < \mu_2$ and $\sigma_1 = \mu_2$



# **Figure 6.4** Normal curves with $\mu_1 = \mu_2$ and $\sigma_1 < \sigma_2$



# **Figure 6.5** Normal curves with $\mu_1 < \mu_2$ and $\sigma_1 < \mu_2$

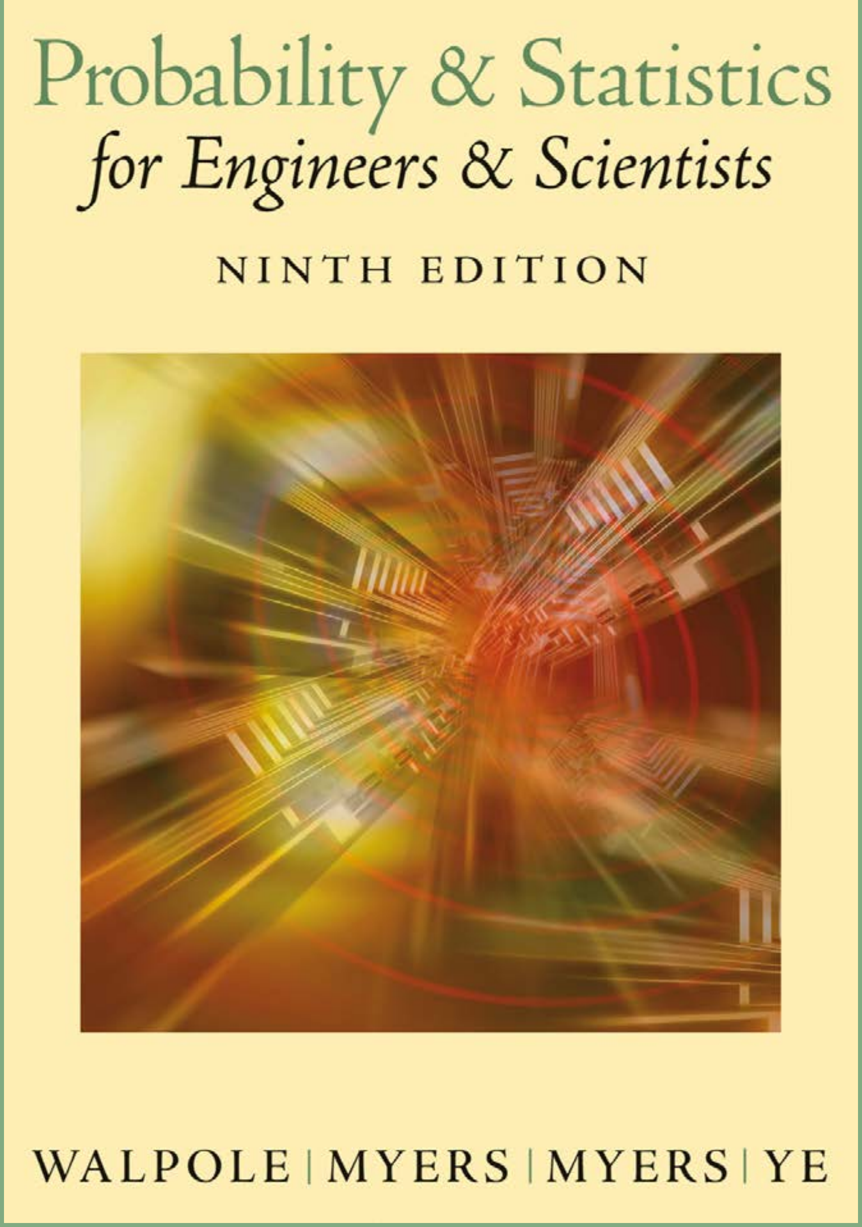


# Theorem 6.2

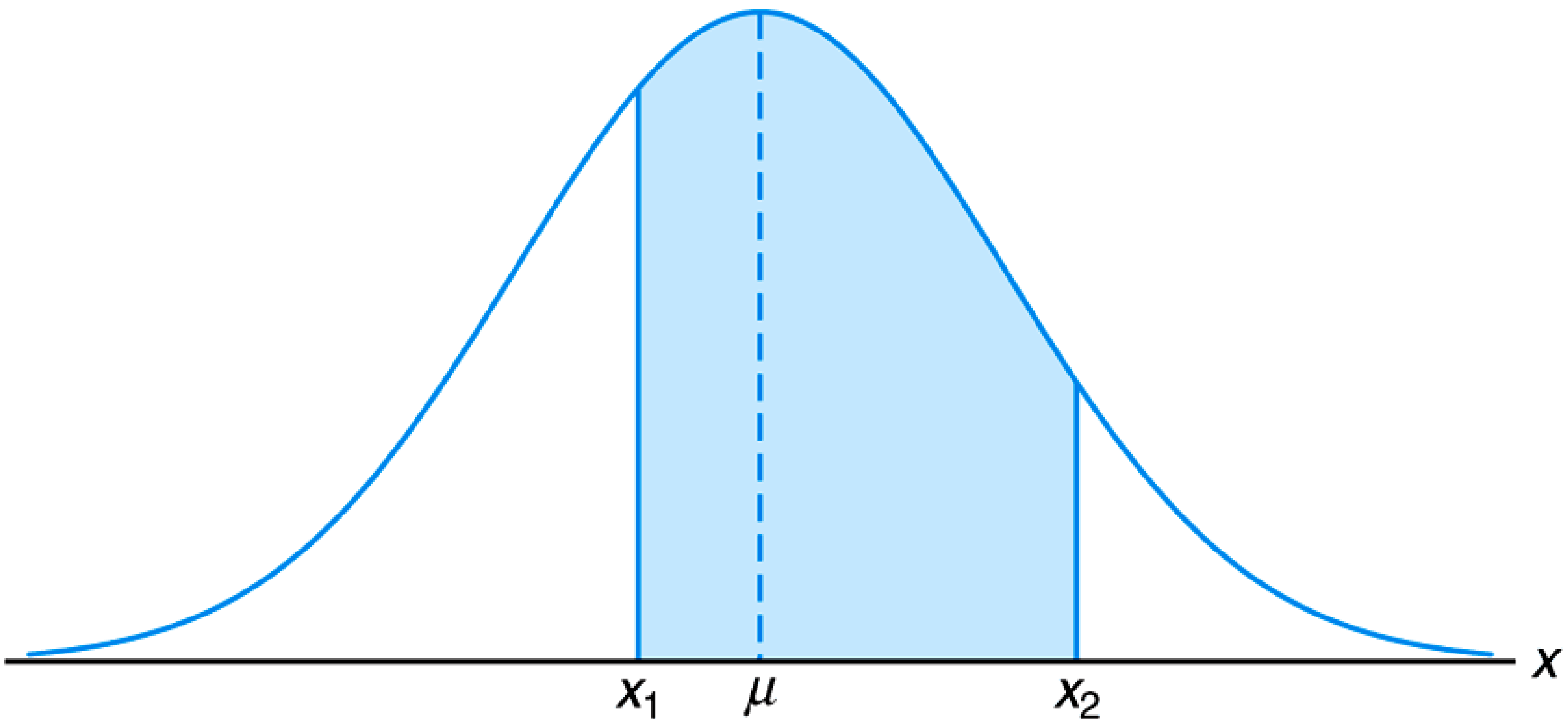
The mean and variance of $n(x; \mu, \sigma)$ are $\mu$ and $\sigma^2$, respectively. Hence, the standard deviation is $\sigma$.

# Section 6.3

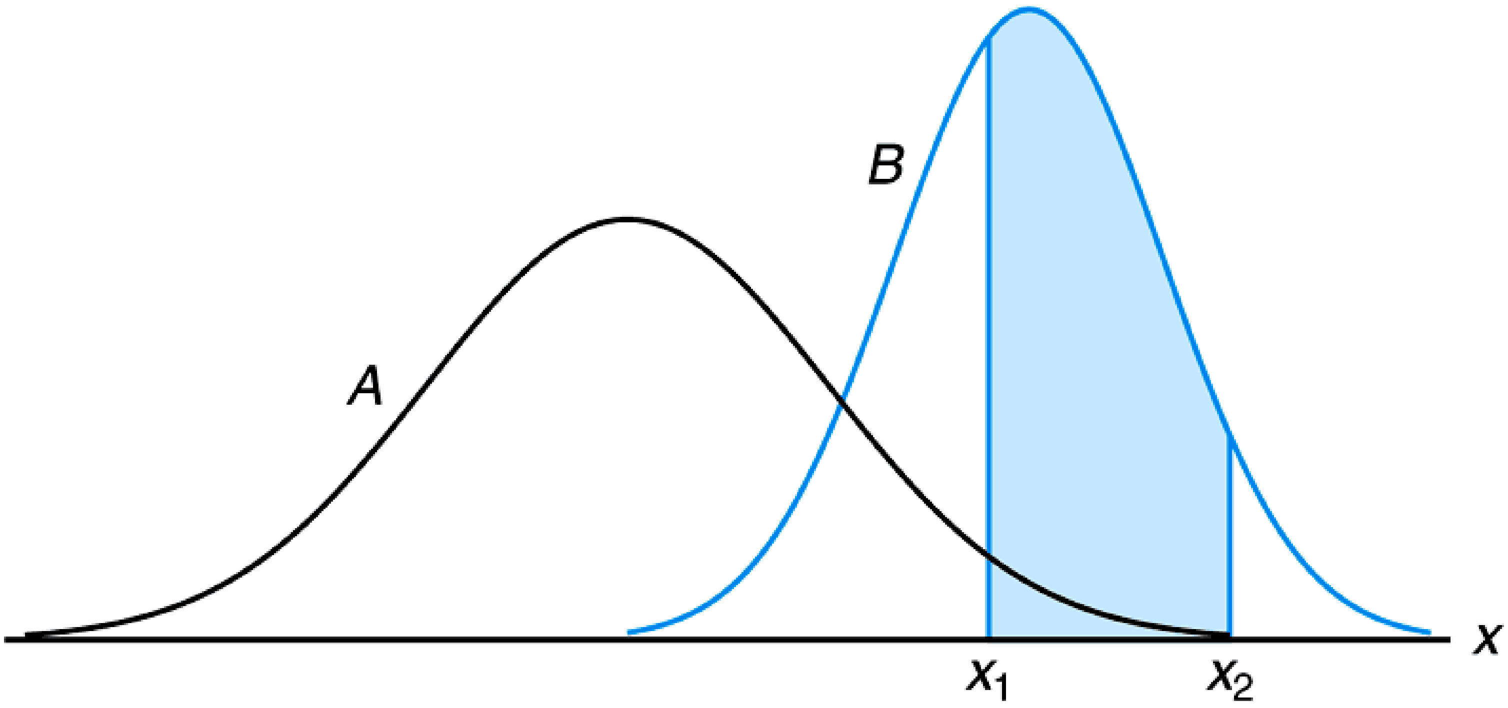
Areas under the Normal Curve



# **Figure 6.6** $P(x_1 < X < x_2)$ = area of the shaded region



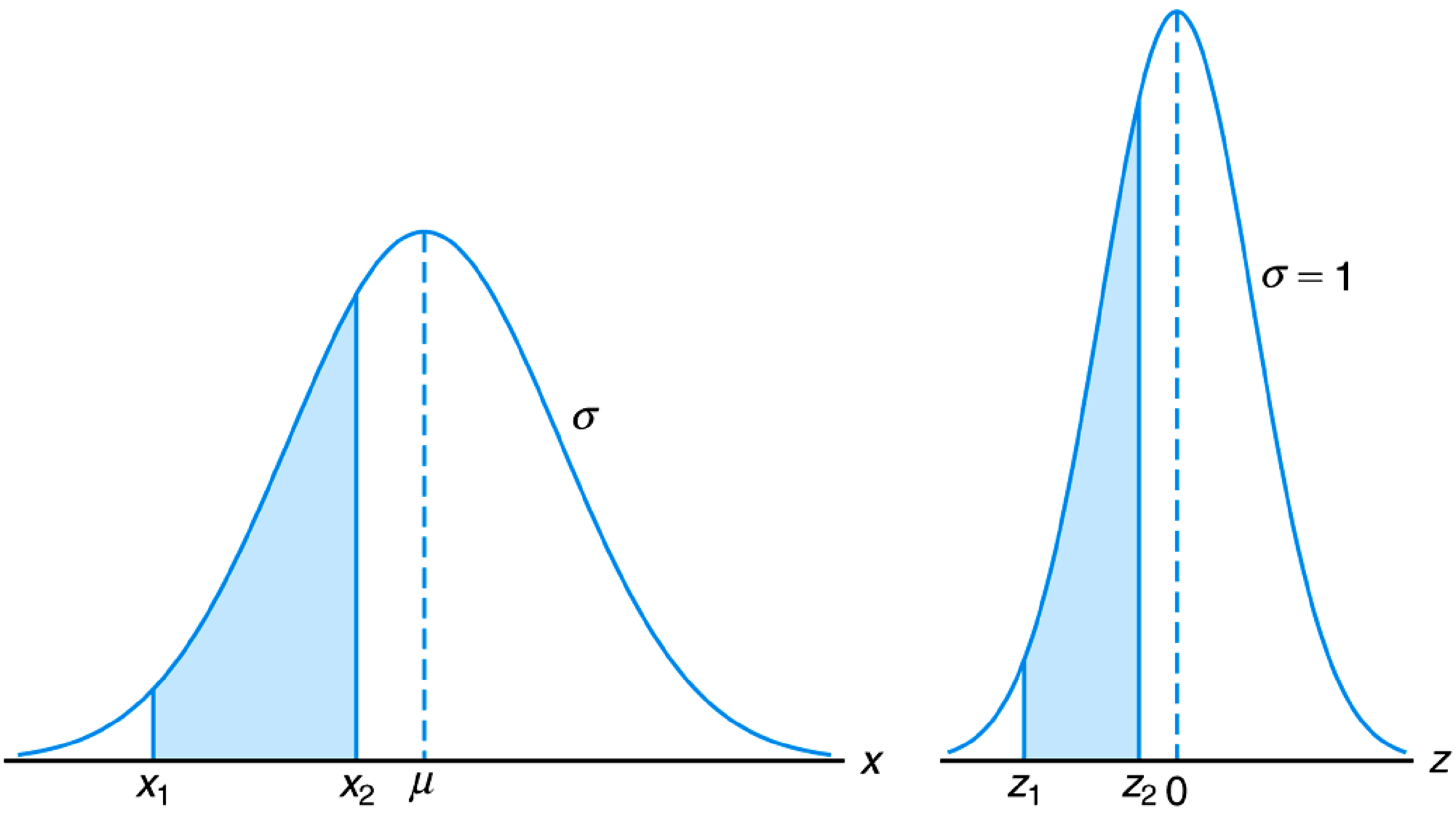
# **Figure 6.7** $P(x_1 < X < x_2)$ for different normal curves



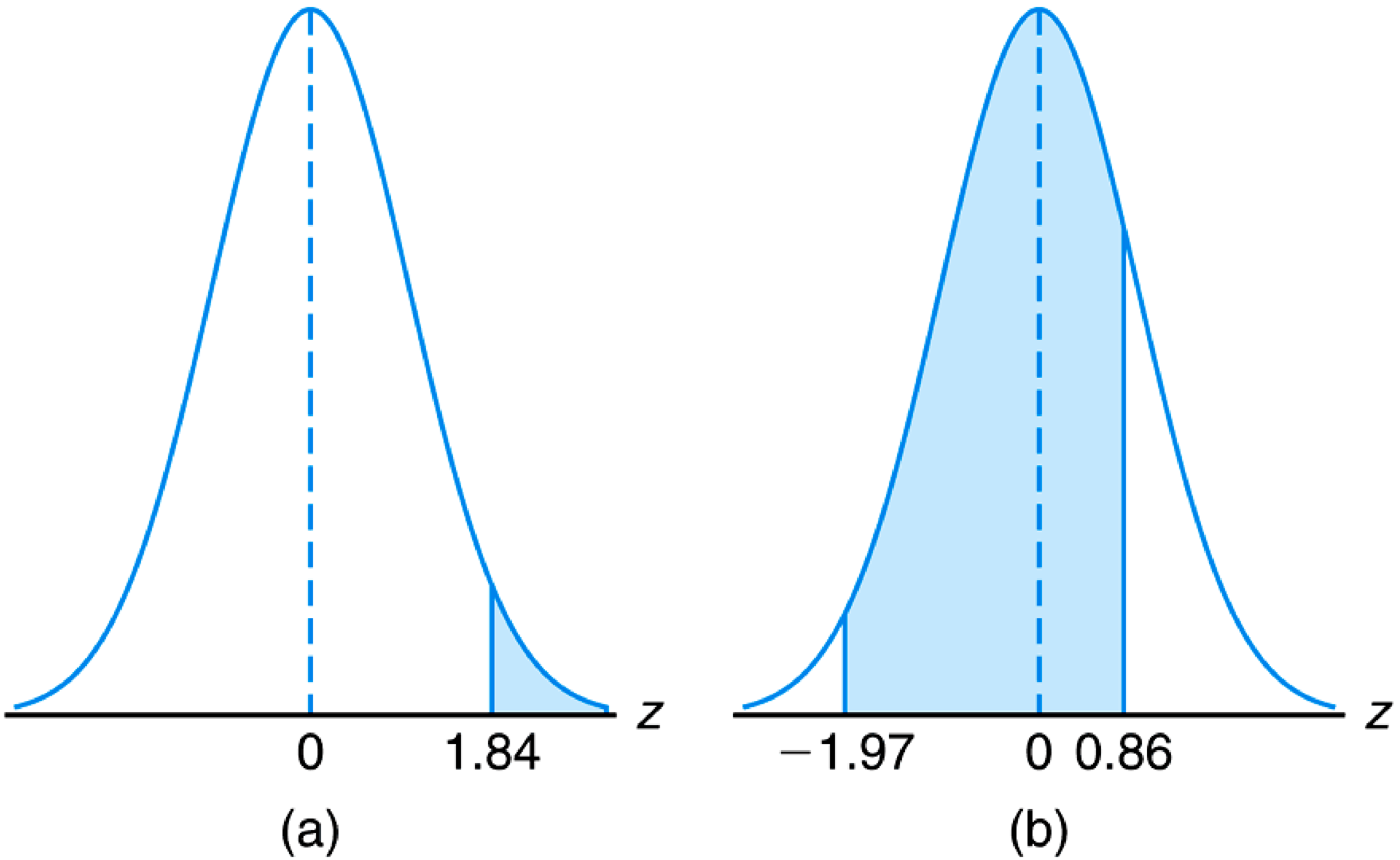
# Definition 6.1

The distribution of a normal random variable with mean 0 and variance 1 is called a **standard normal distribution**.

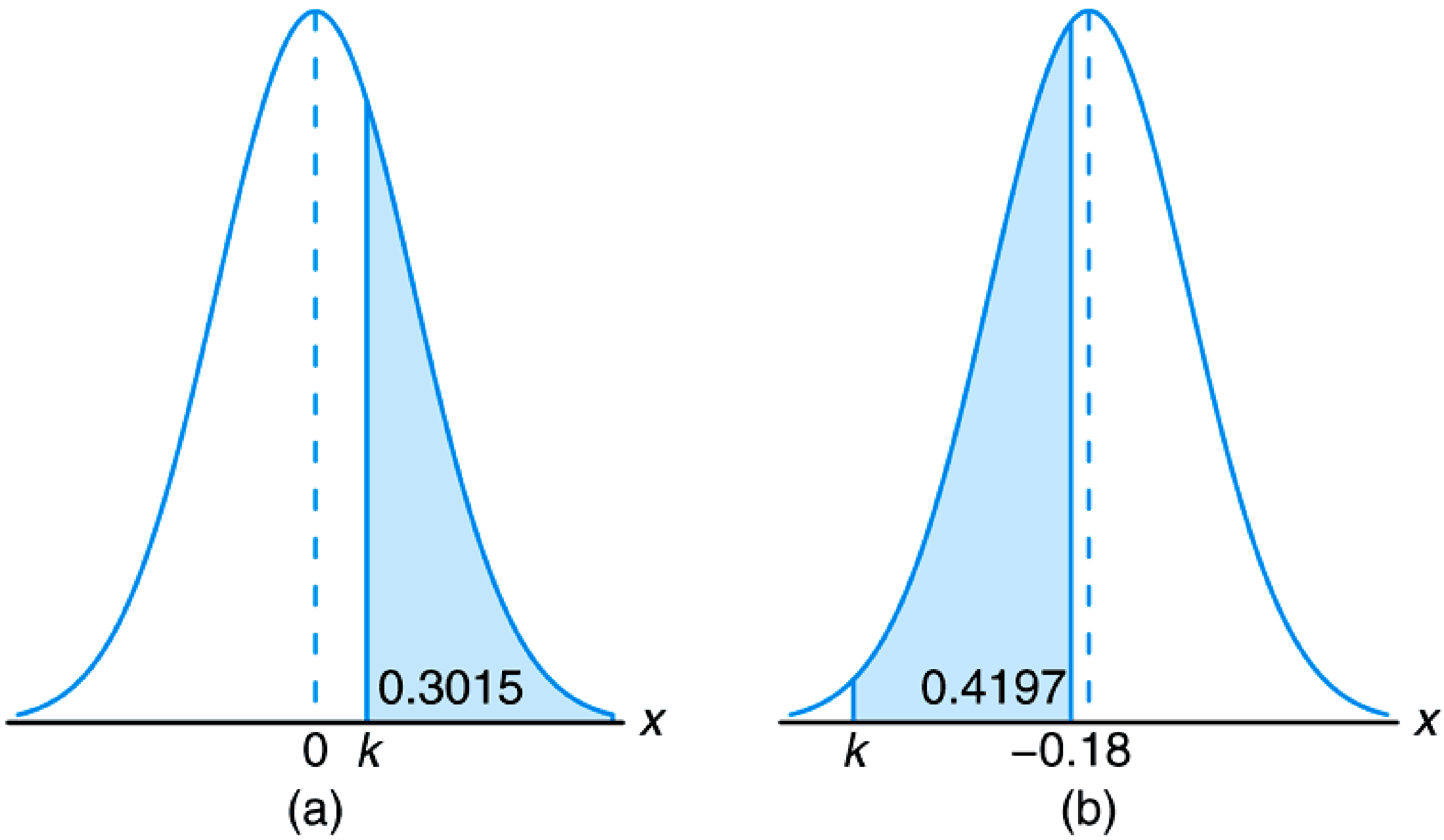
# **Figure 6.8** The original and transformed normal distributions



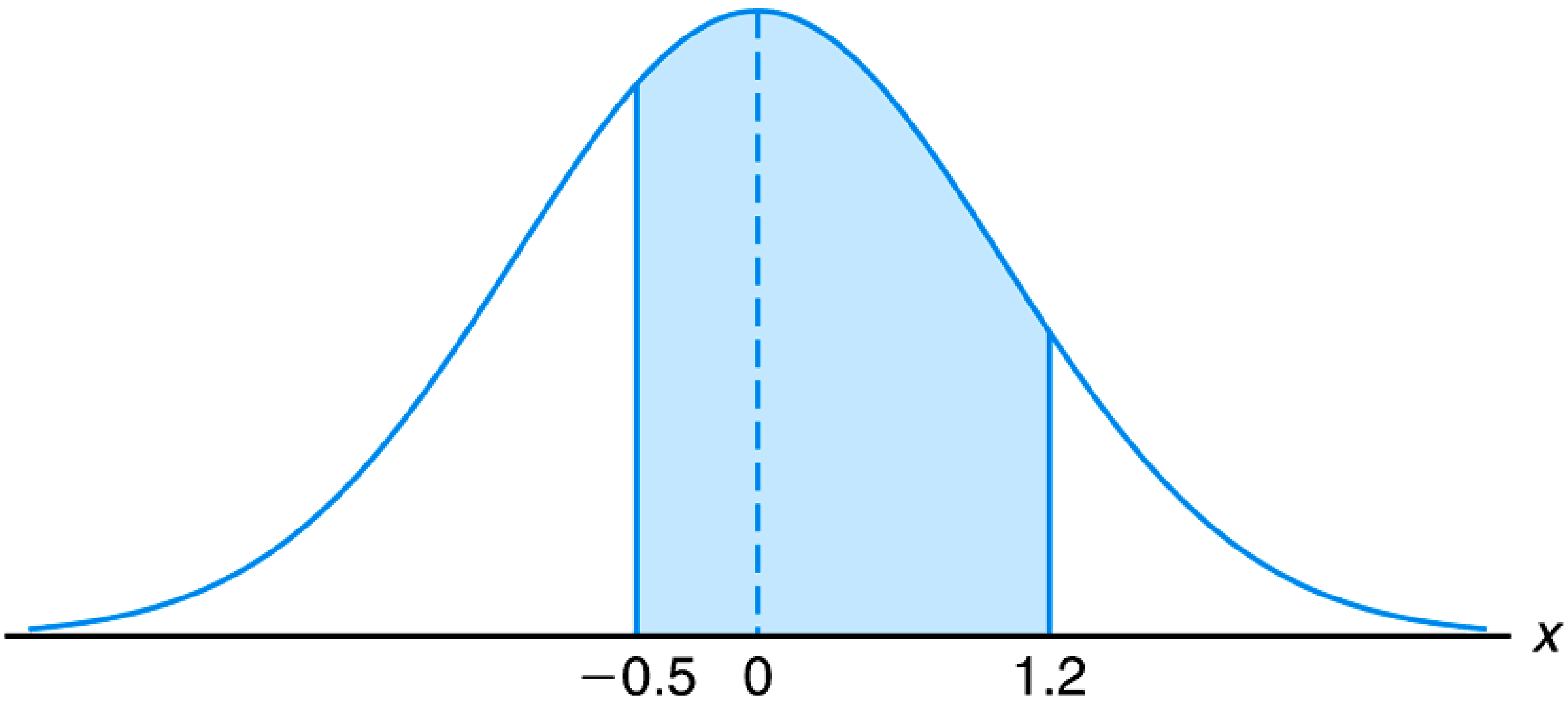
# **Figure 6.9** Areas for Example 6.2



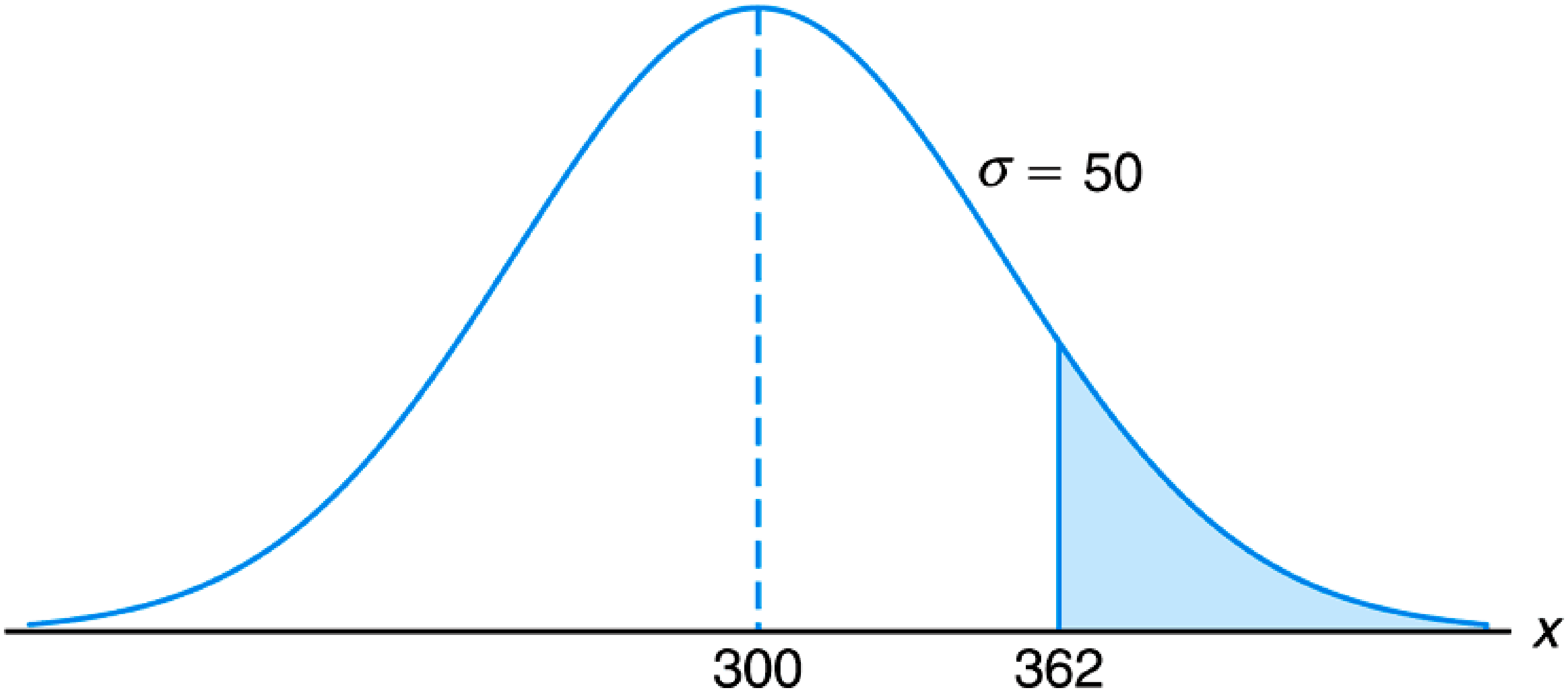
# **Figure 6.10** Areas for Example 6.3



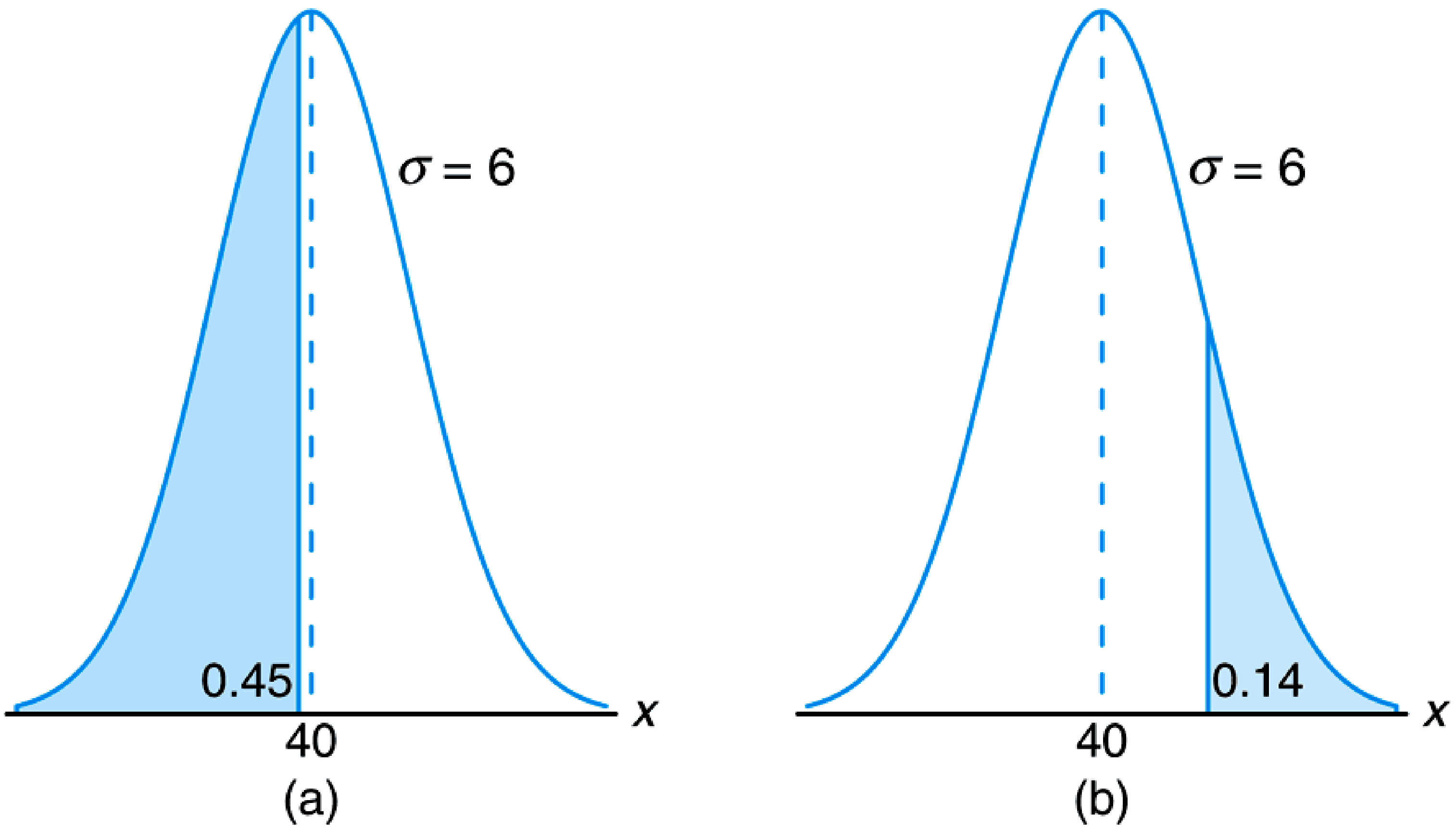
# **Figure 6.11** Area for Example 6.4



# **Figure 6.12** Area for Example 6.5

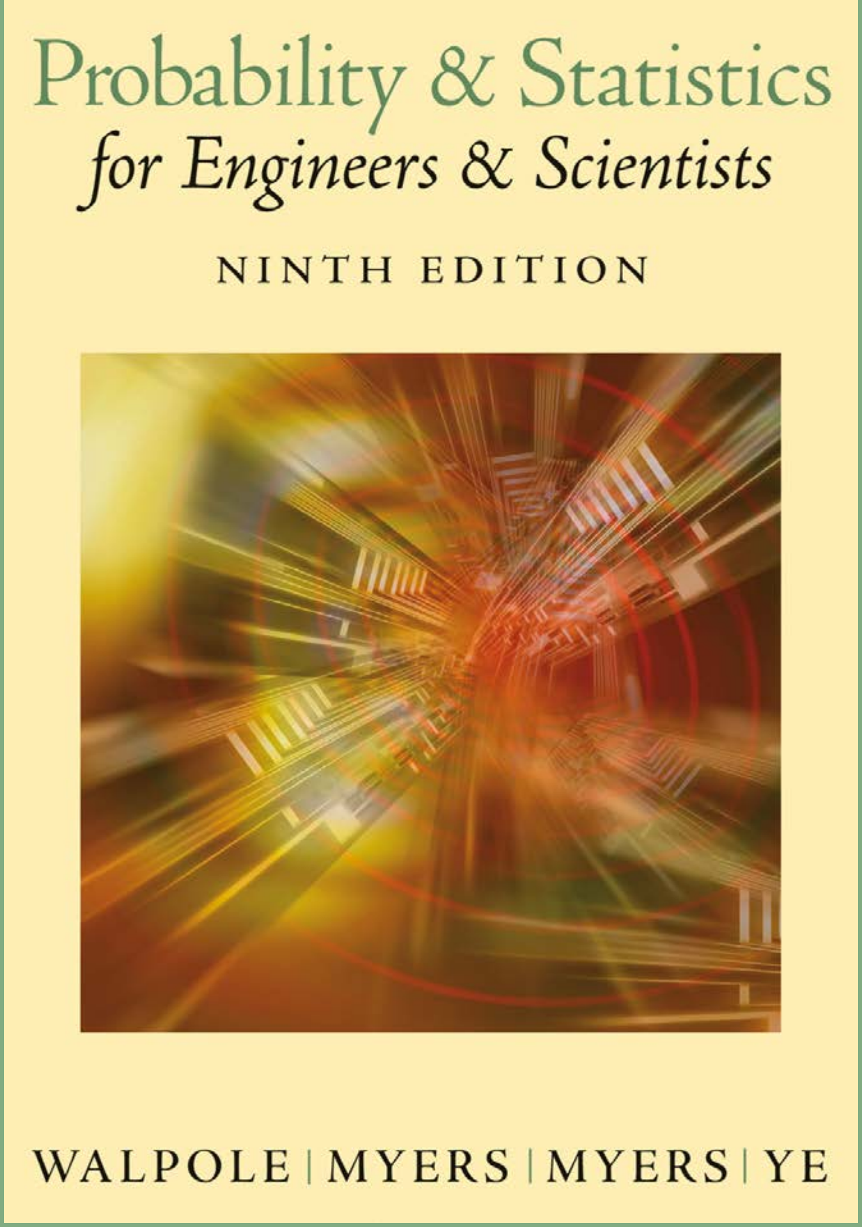


# **Figure 6.13** Areas for Example 6.6

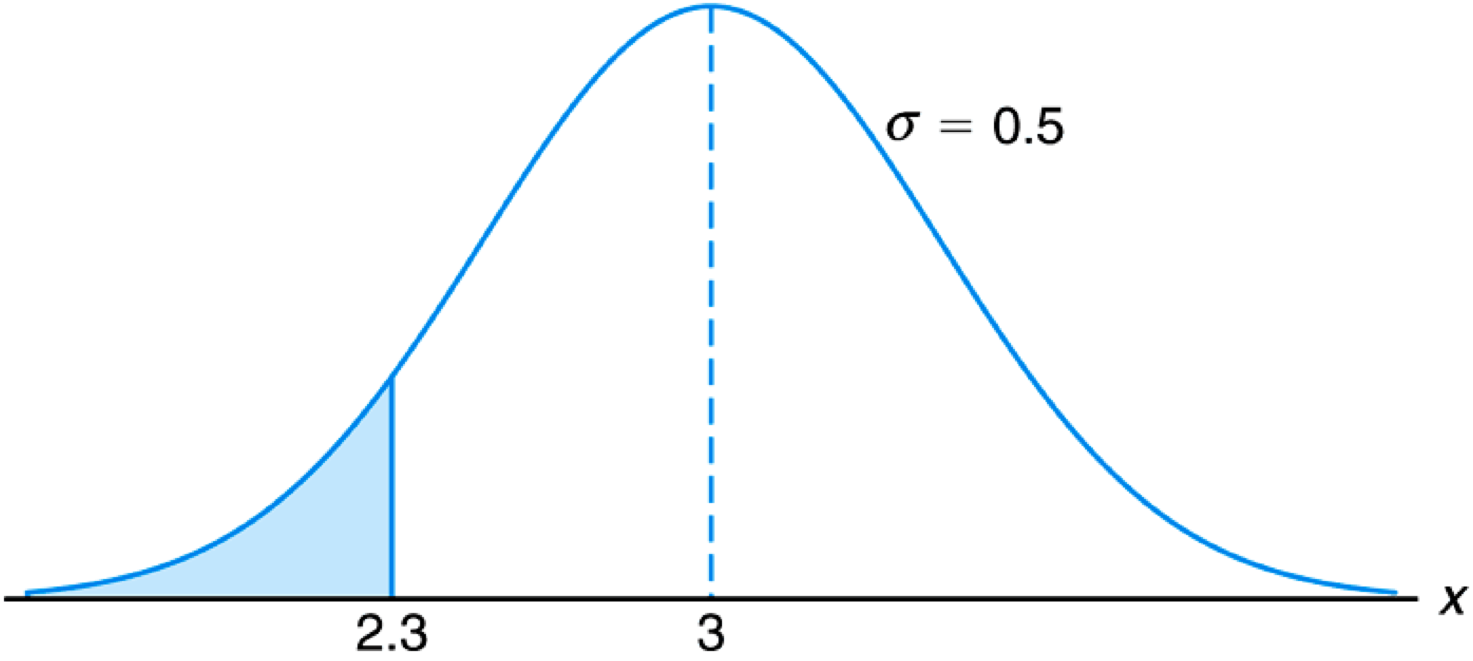


# Section 6.4

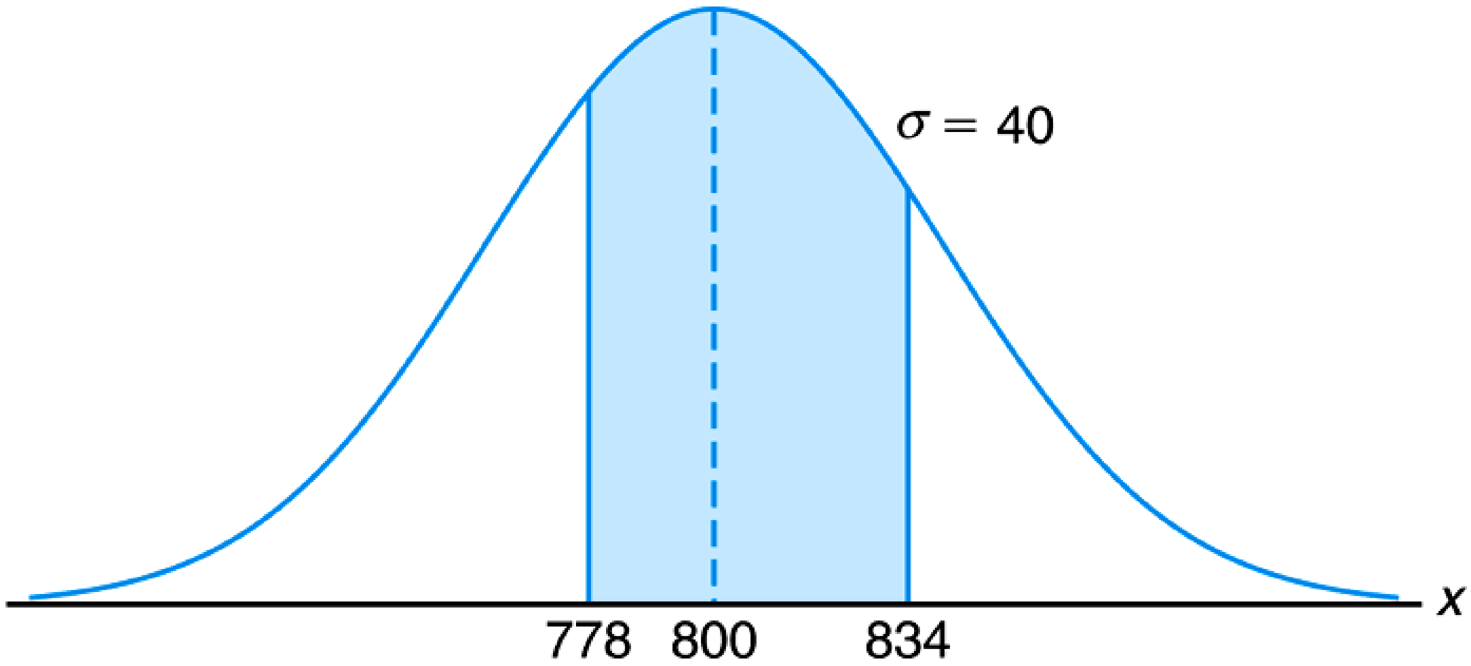
## Applications of the Normal Distribution



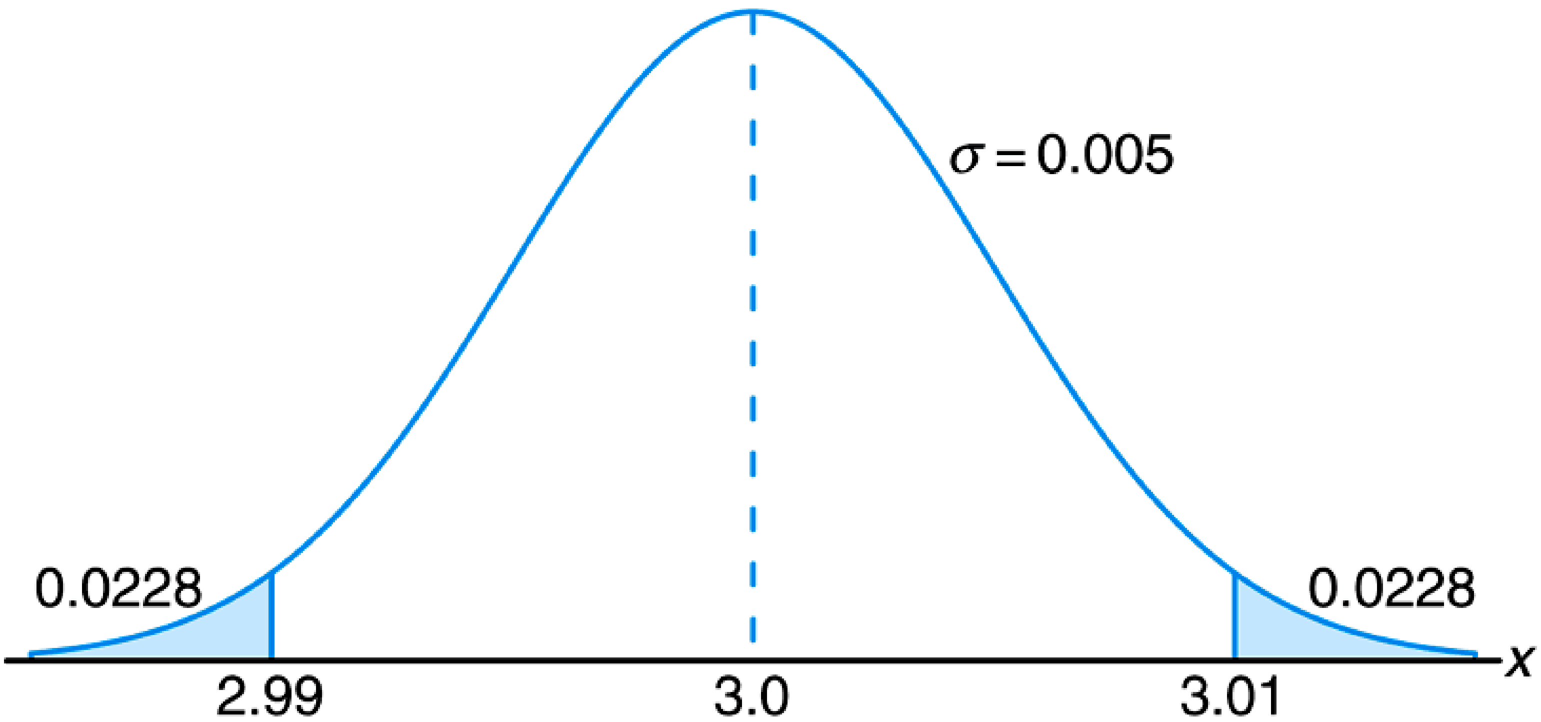
# **Figure 6.14** Area for Example 6.7



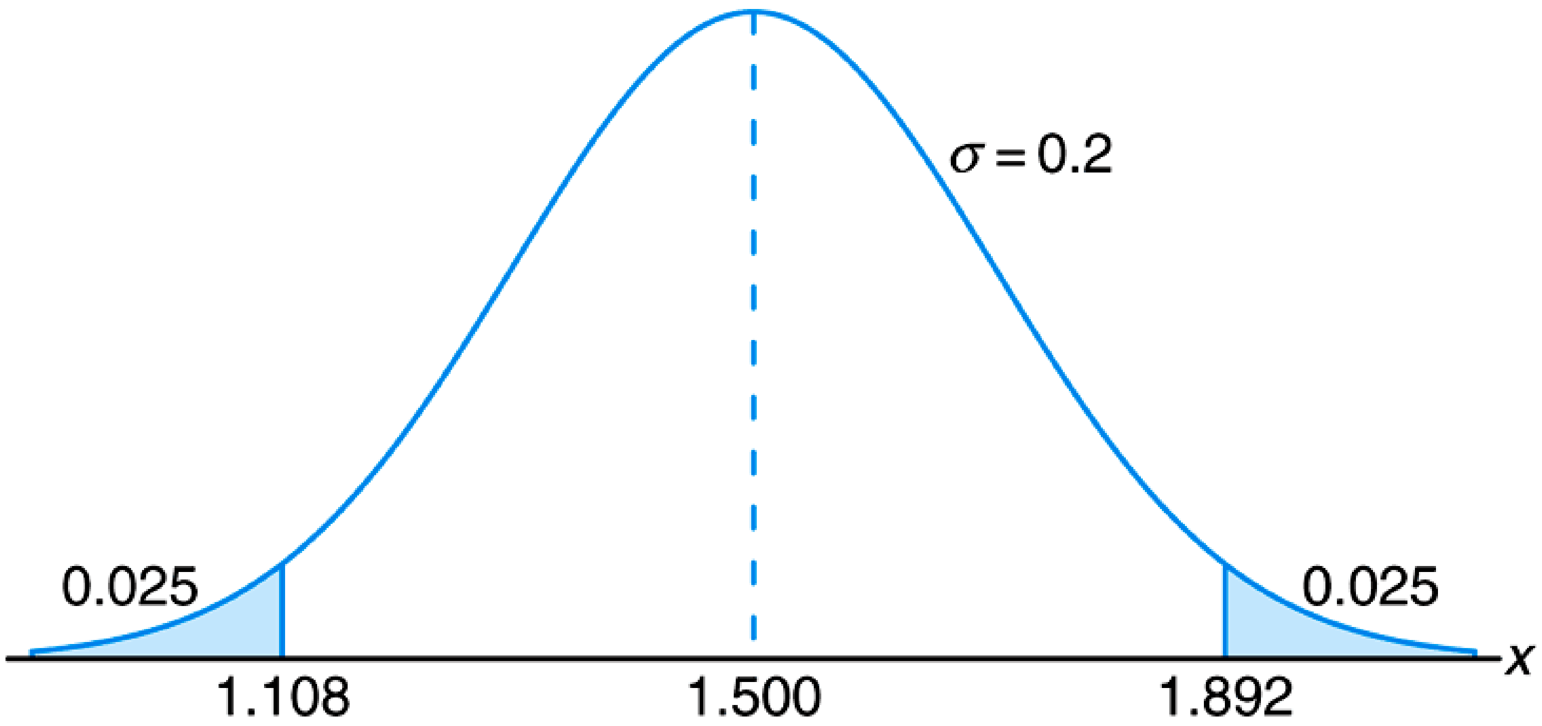
# **Figure 6.15** Area for Example 6.8



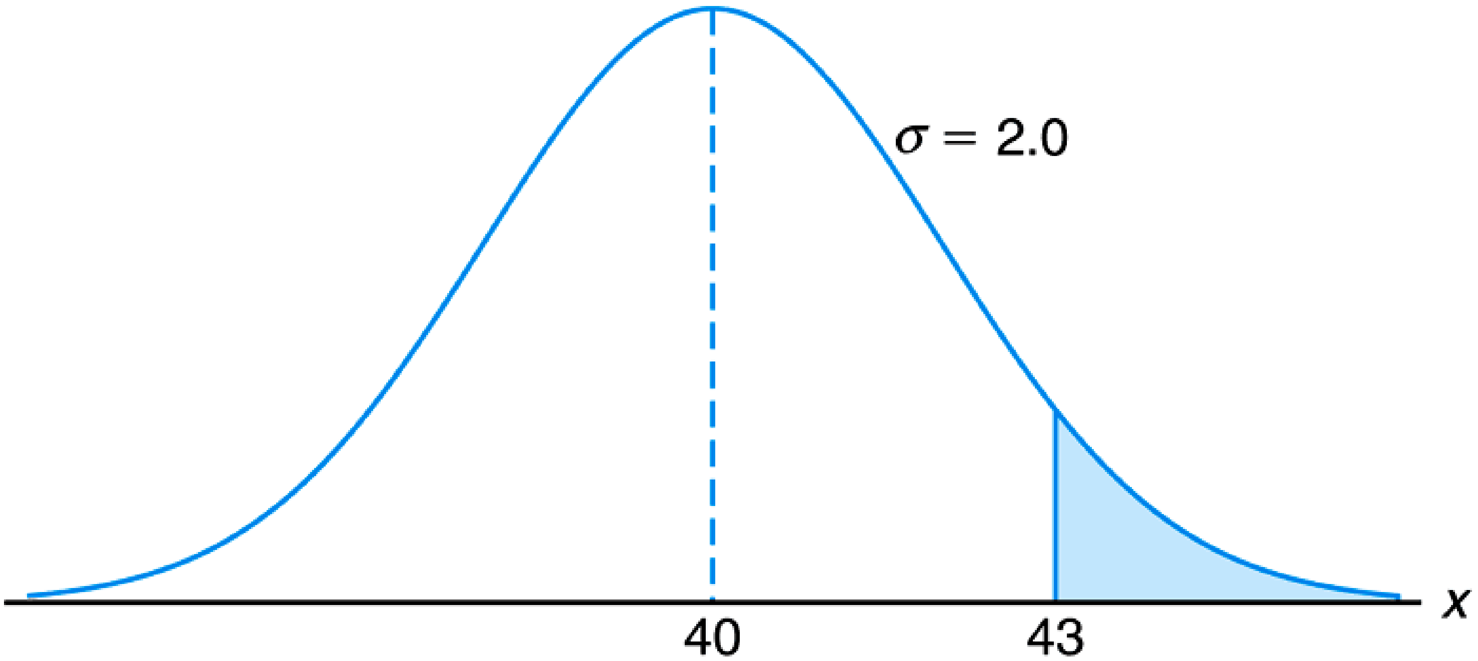
# **Figure 6.16** Area for Example 6.9



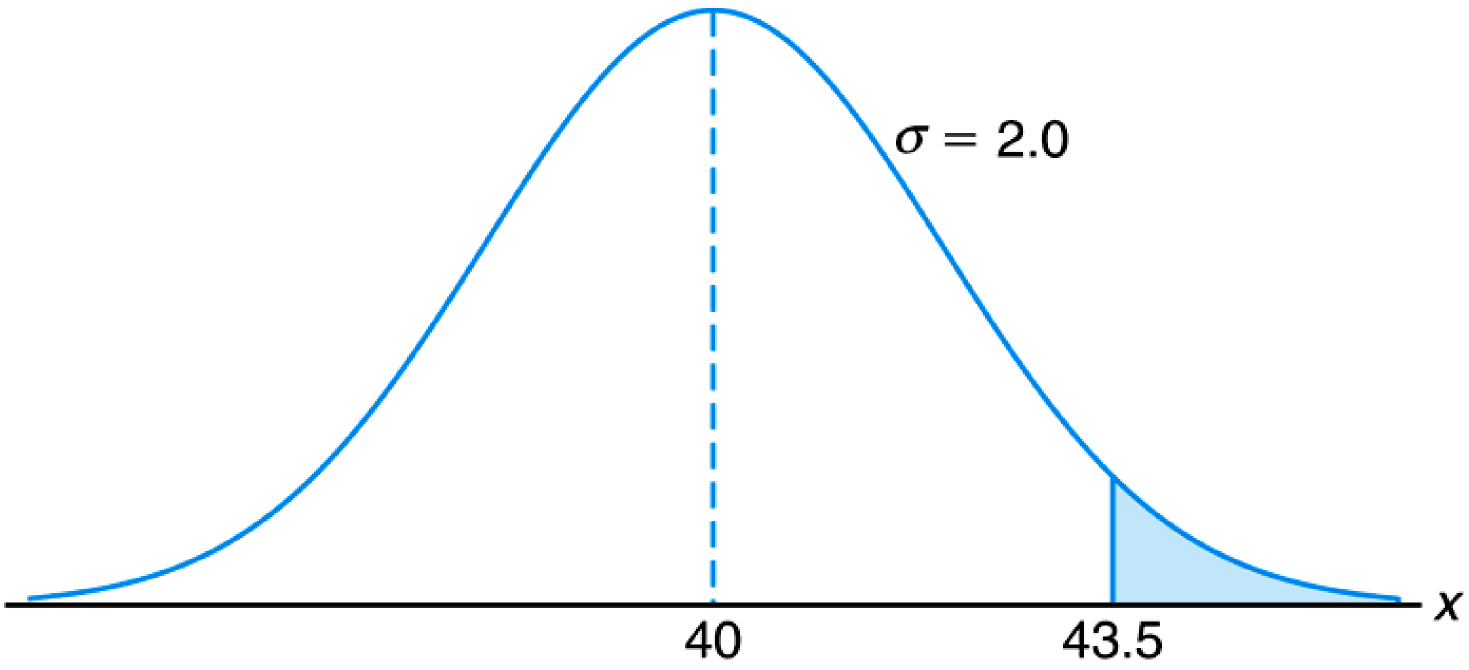
# **Figure 6.17** Specifications for Example 6.10



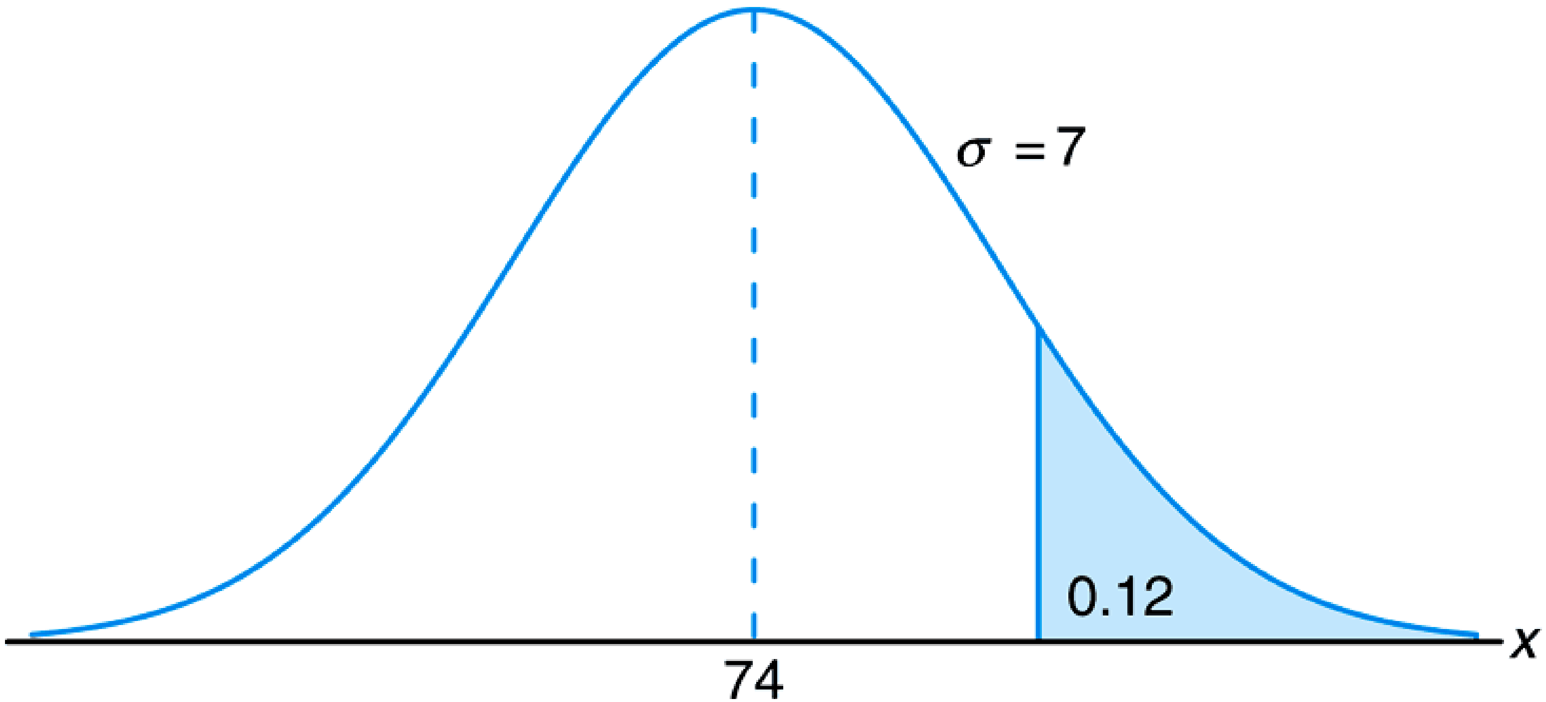
# **Figure 6.18** Area for Example 6.11



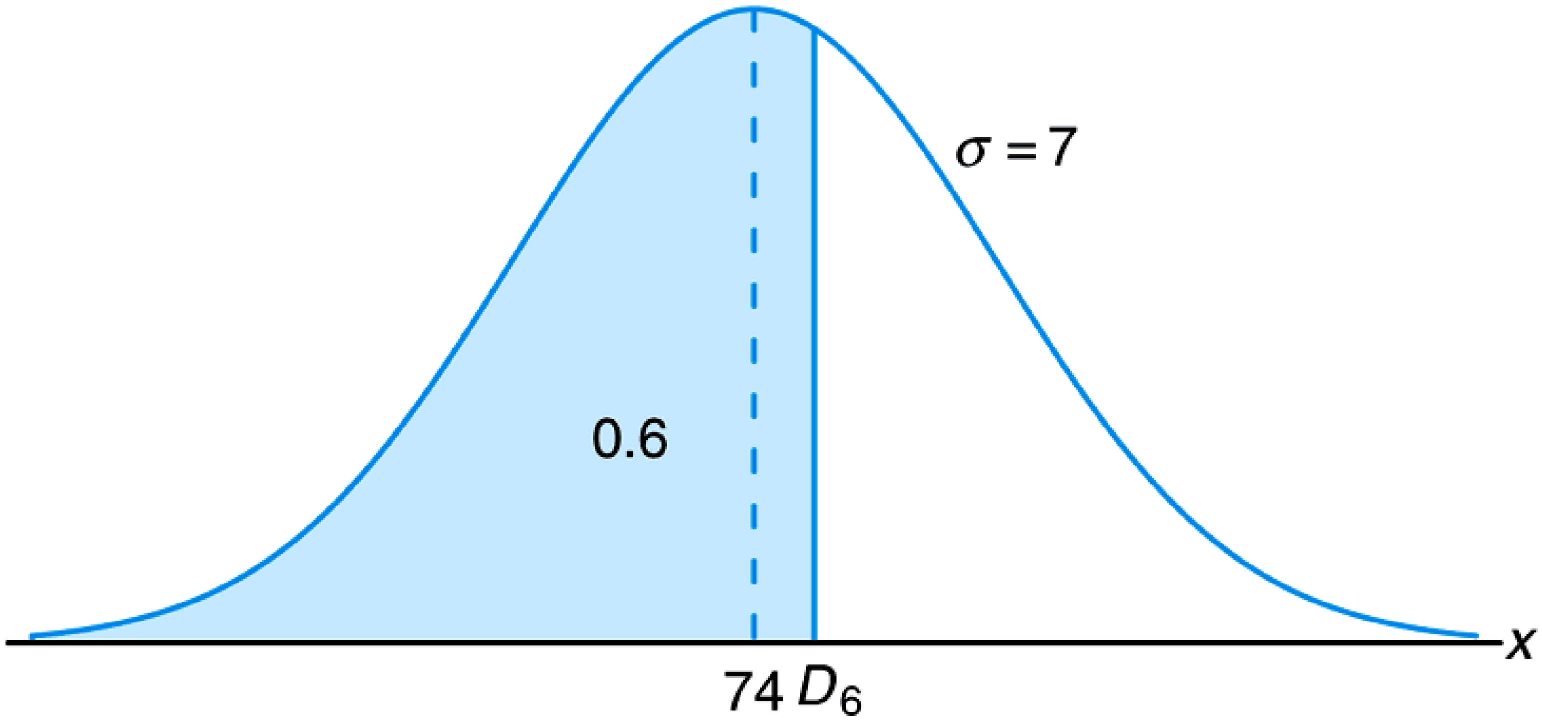
# **Figure 6.19** Area for Example 6.12



# **Figure 6.20** Area for Example 6.13

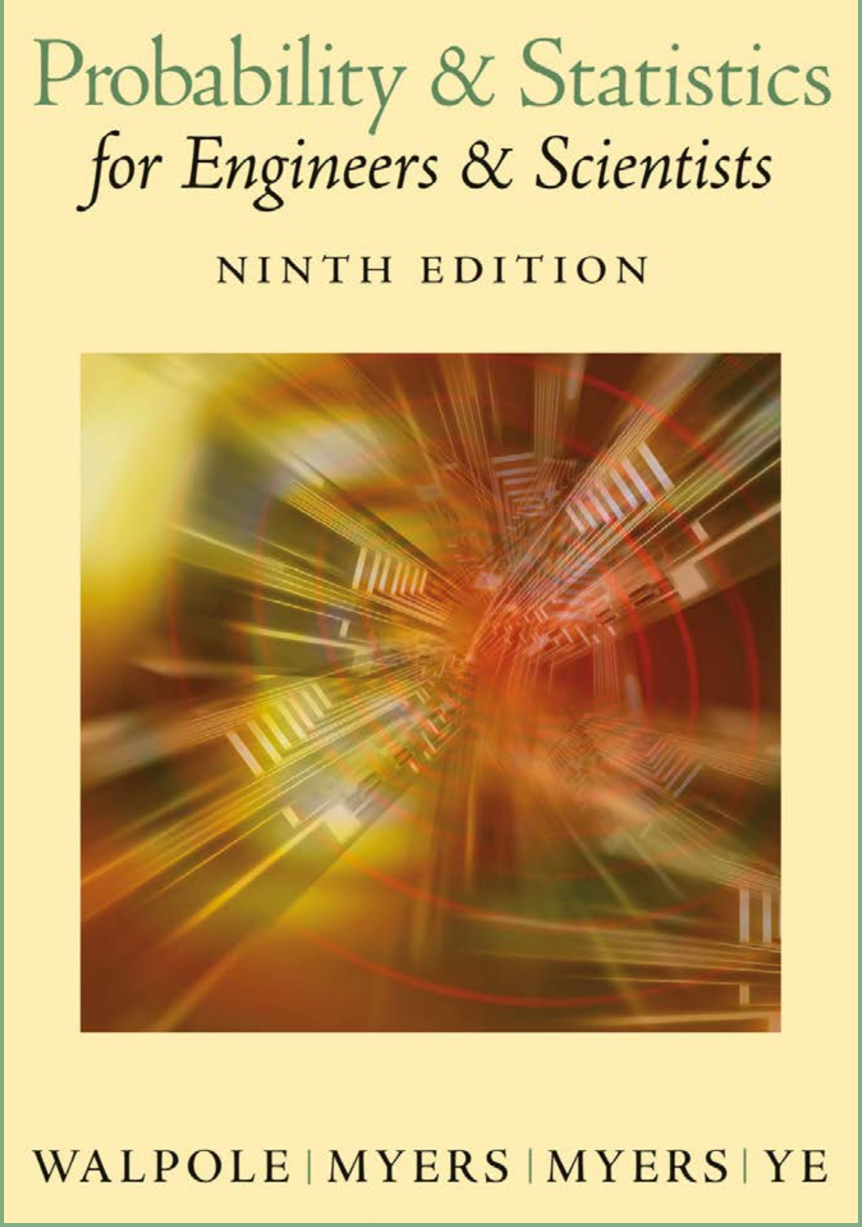


# **Figure 6.21** Area for Example 6.14



# Section 6.5

Normal Approximation to the Binomial



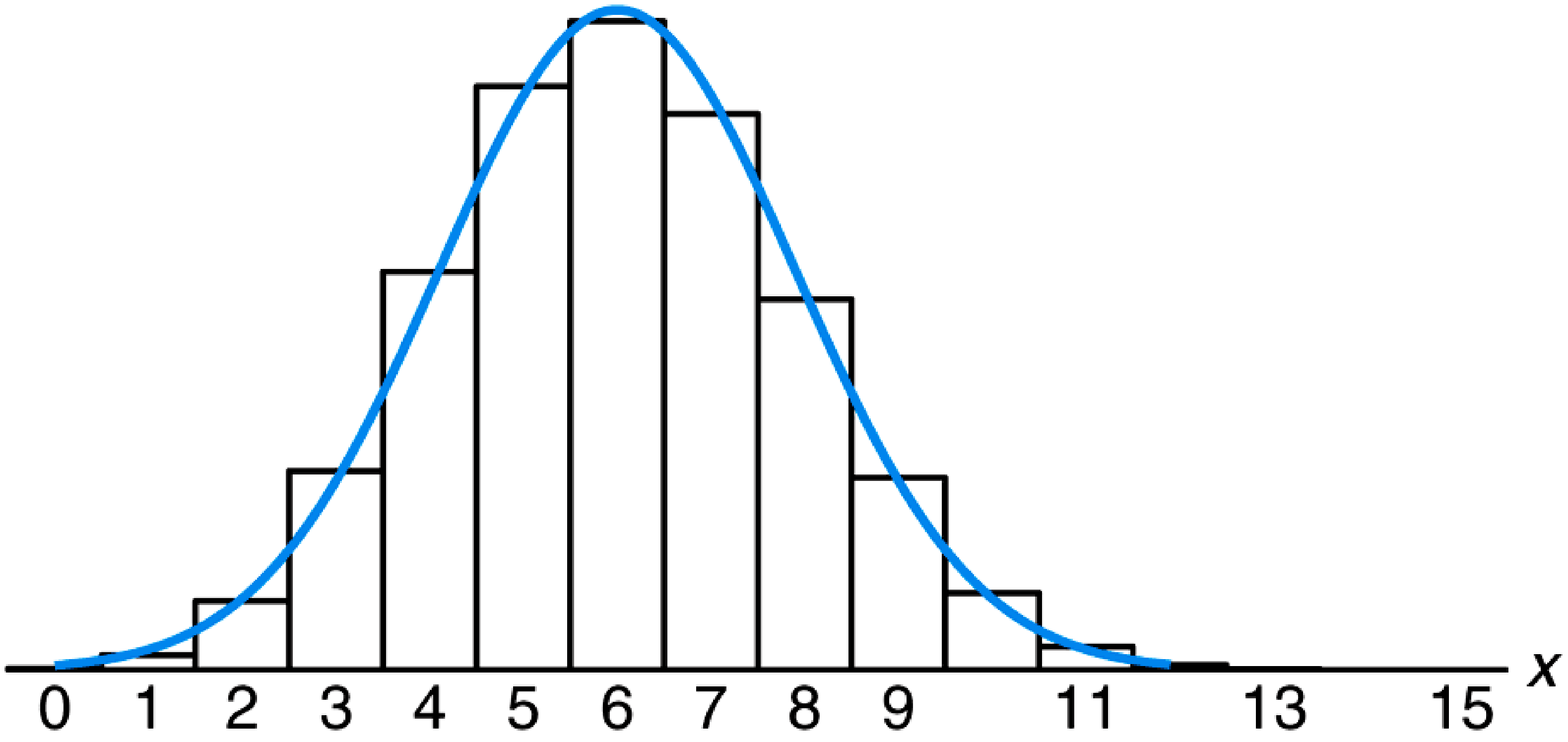
# Theorem 6.3

If $X$ is a binomial random variable with mean $\mu = np$ and variance $\sigma^2 = npq$, then the limiting form of the distribution of

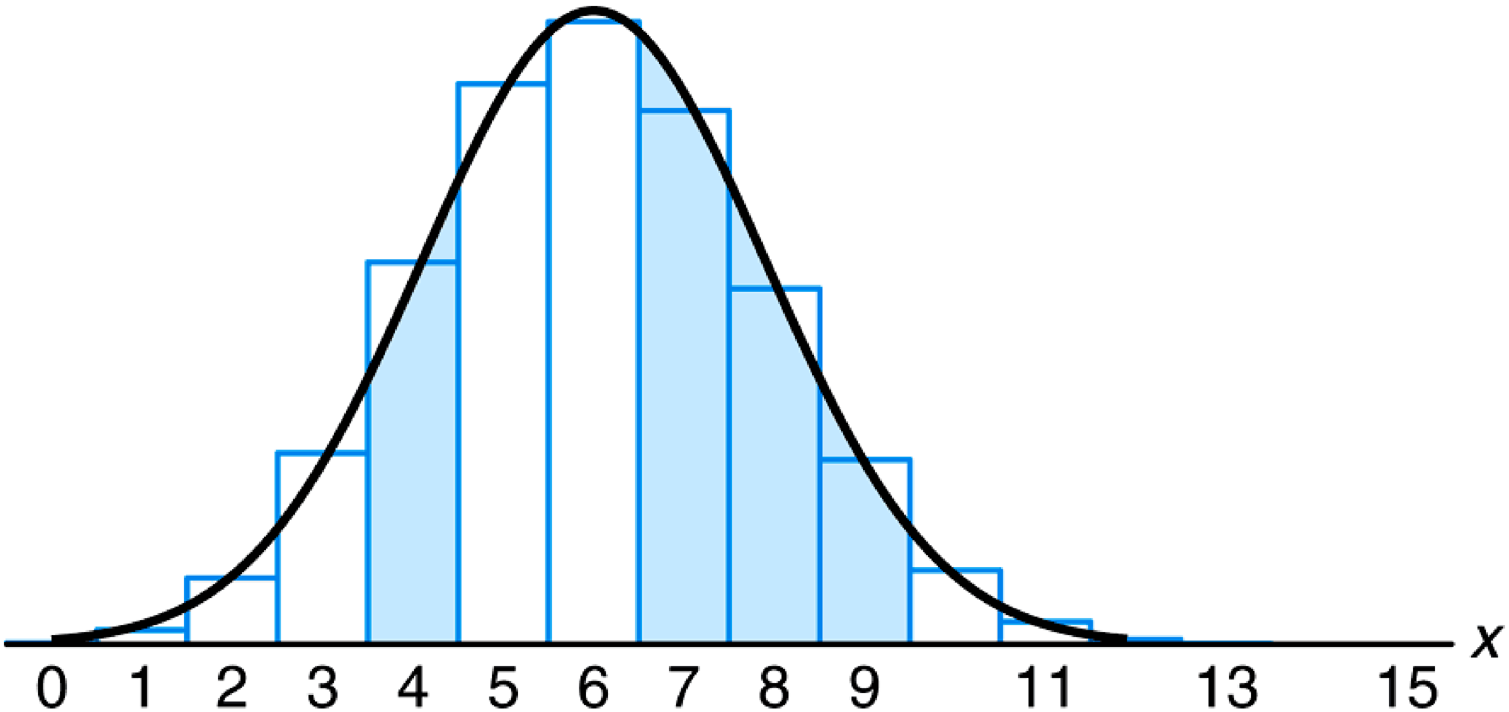
$$Z = \frac{X - np}{\sqrt{npq}},$$

as $n \to \infty$, is the standard normal distribution $n(z; 0, 1)$.

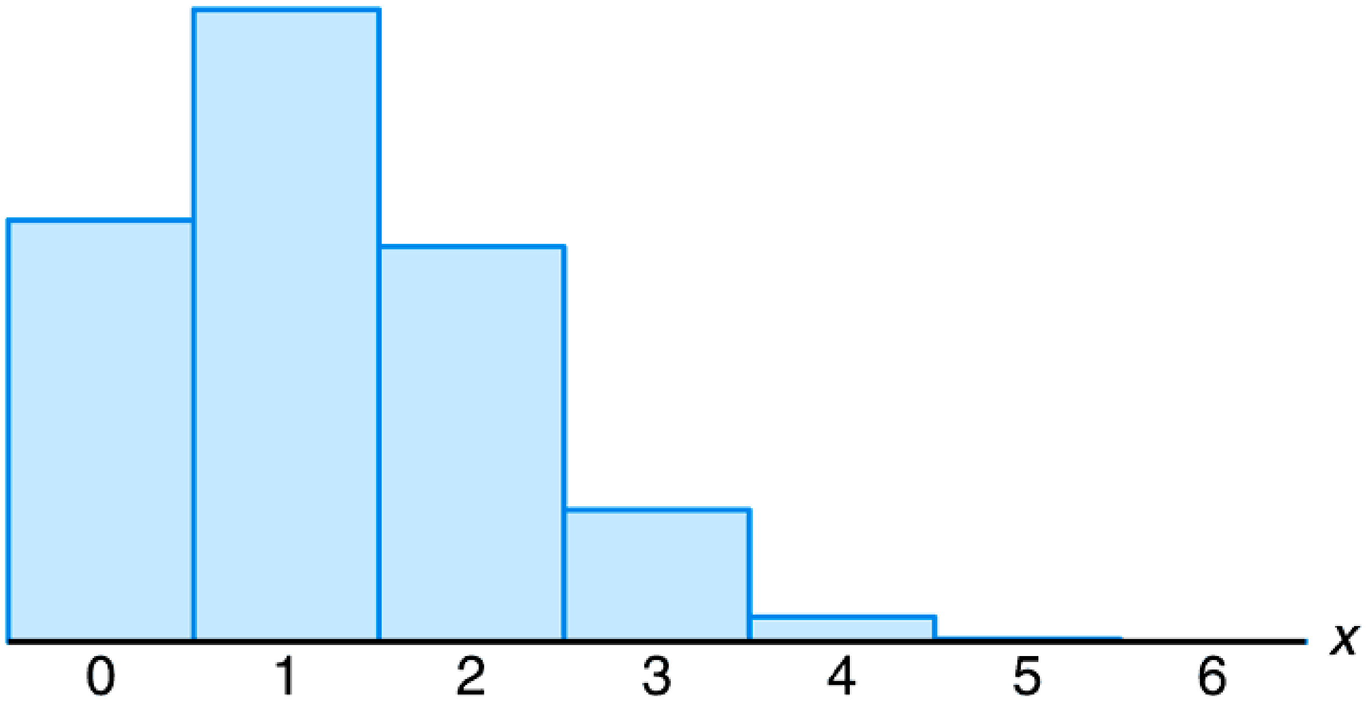
# **Figure 6.22** Normal approximation of $b(x; 15, 0.4)$



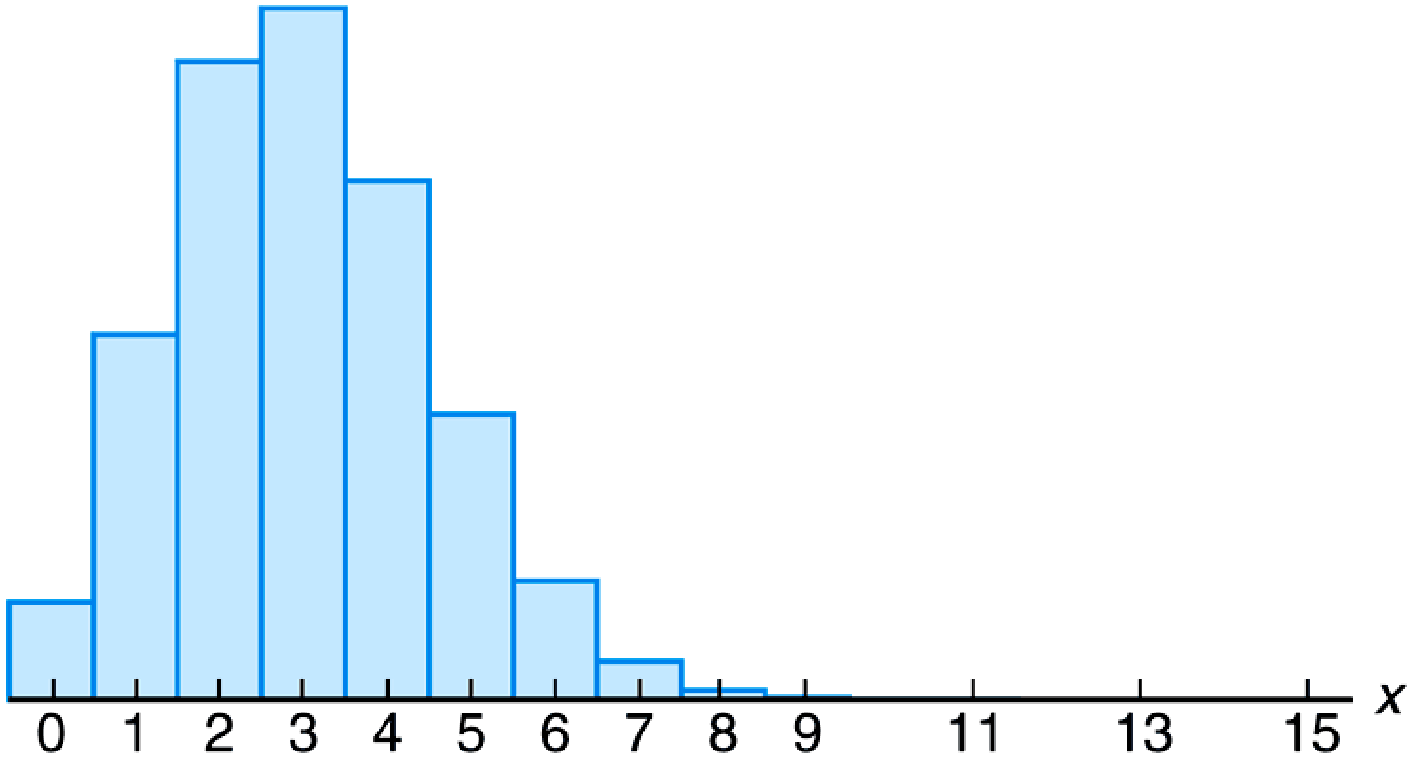
# **Figure 6.23** Normal approximation of $b(x;\,15,\,0.4)$ and $\sum_{x=7}^{9} b(x;\,15,\,0.4)$



# **Figure 6.24** Histogram for $b(x; 6, 0.2)$



# **Figure 6.25** Histogram for $b(x; 15, 0.2)$

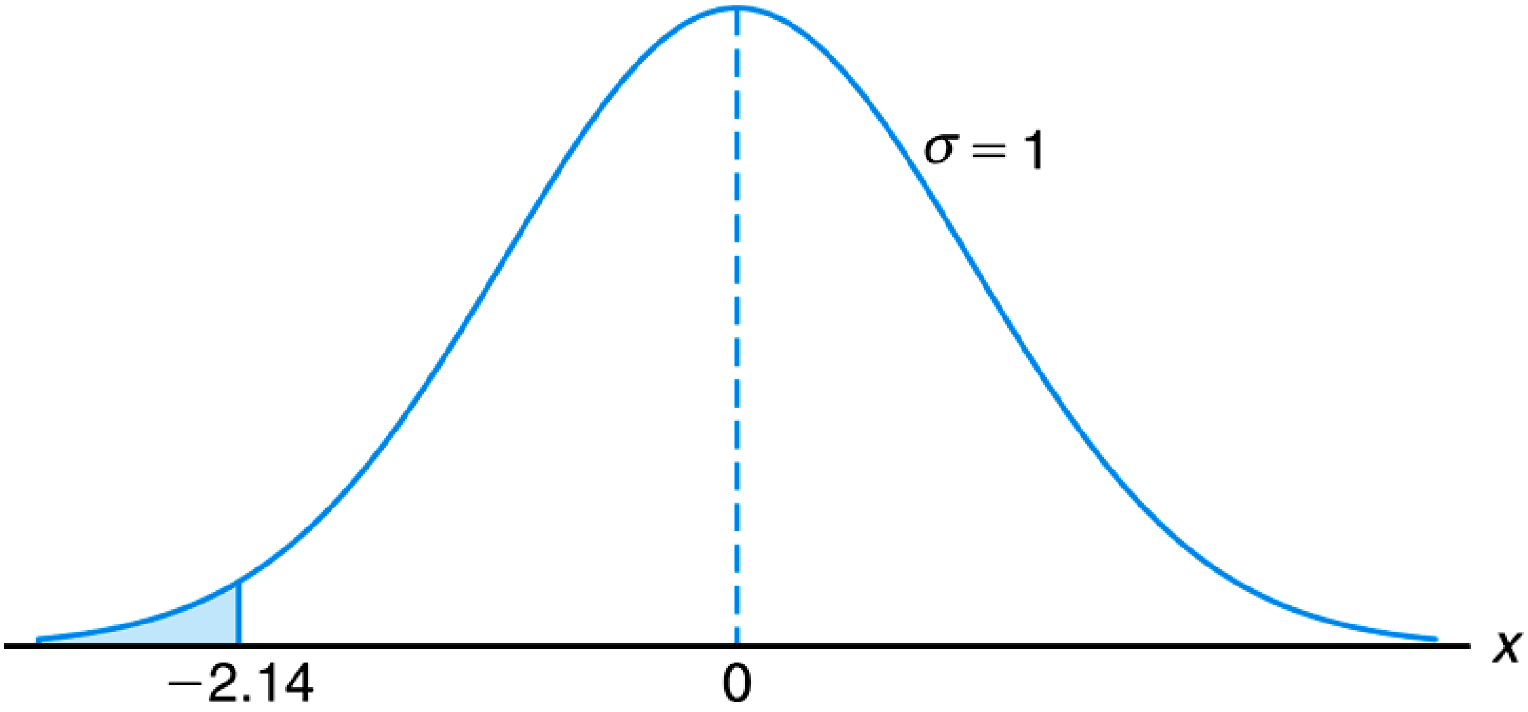


# **Table 6.1** Normal Approximation and True Cumulative Binomial Probabilities

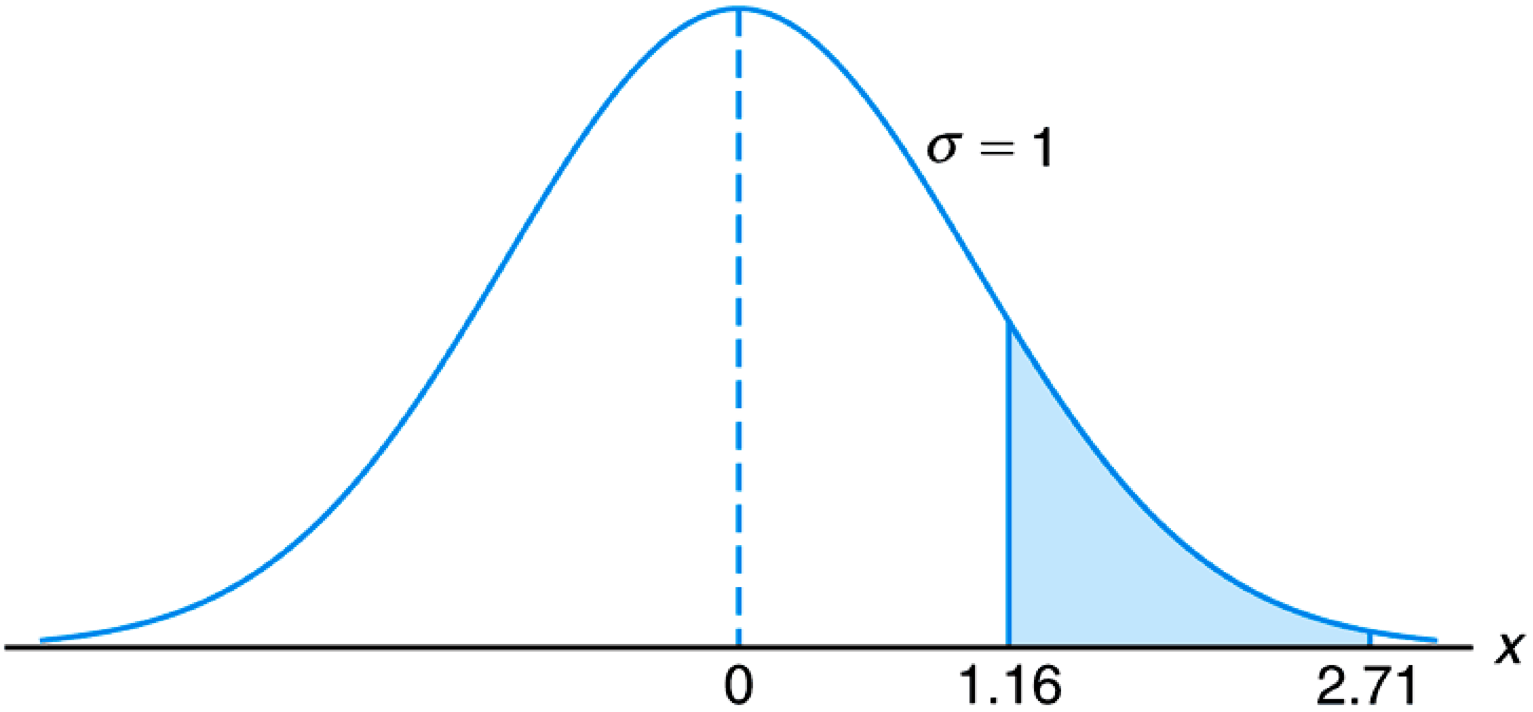
| | $p = 0.05,\ n = 10$ | | $p = 0.10,\ n = 10$ | | $p = 0.50,\ n = 10$ | |
|---|---|---|---|---|---|---|
| $r$ | **Binomial** | **Normal** | **Binomial** | **Normal** | **Binomial** | **Normal** |
| 0 | 0.5987 | 0.5000 | 0.3487 | 0.2981 | 0.0010 | 0.0022 |
| 1 | 0.9139 | 0.9265 | 0.7361 | 0.7019 | 0.0107 | 0.0136 |
| 2 | 0.9885 | 0.9981 | 0.9298 | 0.9429 | 0.0547 | 0.0571 |
| 3 | 0.9990 | 1.0000 | 0.9872 | 0.9959 | 0.1719 | 0.1711 |
| 4 | 1.0000 | 1.0000 | 0.9984 | 0.9999 | 0.3770 | 0.3745 |
| 5 | | | 1.0000 | 1.0000 | 0.6230 | 0.6255 |
| 6 | | | | | 0.8281 | 0.8289 |
| 7 | | | | | 0.9453 | 0.9429 |
| 8 | | | | | 0.9893 | 0.9864 |
| 9 | | | | | 0.9990 | 0.9978 |
| 10 | | | | | 1.0000 | 0.9997 |

| | $p = 0.05$ | | | | | |
|---|---|---|---|---|---|---|
| | $n = 20$ | | $n = 50$ | | $n = 100$ | |
| $r$ | **Binomial** | **Normal** | **Binomial** | **Normal** | **Binomial** | **Normal** |
| 0 | 0.3585 | 0.3015 | 0.0769 | 0.0968 | 0.0059 | 0.0197 |
| 1 | 0.7358 | 0.6985 | 0.2794 | 0.2578 | 0.0371 | 0.0537 |
| 2 | 0.9245 | 0.9382 | 0.5405 | 0.5000 | 0.1183 | 0.1251 |
| 3 | 0.9841 | 0.9948 | 0.7604 | 0.7422 | 0.2578 | 0.2451 |
| 4 | 0.9974 | 0.9998 | 0.8964 | 0.9032 | 0.4360 | 0.4090 |
| 5 | 0.9997 | 1.0000 | 0.9622 | 0.9744 | 0.6160 | 0.5910 |
| 6 | 1.0000 | 1.0000 | 0.9882 | 0.9953 | 0.7660 | 0.7549 |
| 7 | | | 0.9968 | 0.9994 | 0.8720 | 0.8749 |
| 8 | | | 0.9992 | 0.9999 | 0.9369 | 0.9463 |
| 9 | | | 0.9998 | 1.0000 | 0.9718 | 0.9803 |
| 10 | | | 1.0000 | 1.0000 | 0.9885 | 0.9941 |

# **Figure 6.26** Area for Example 6.15

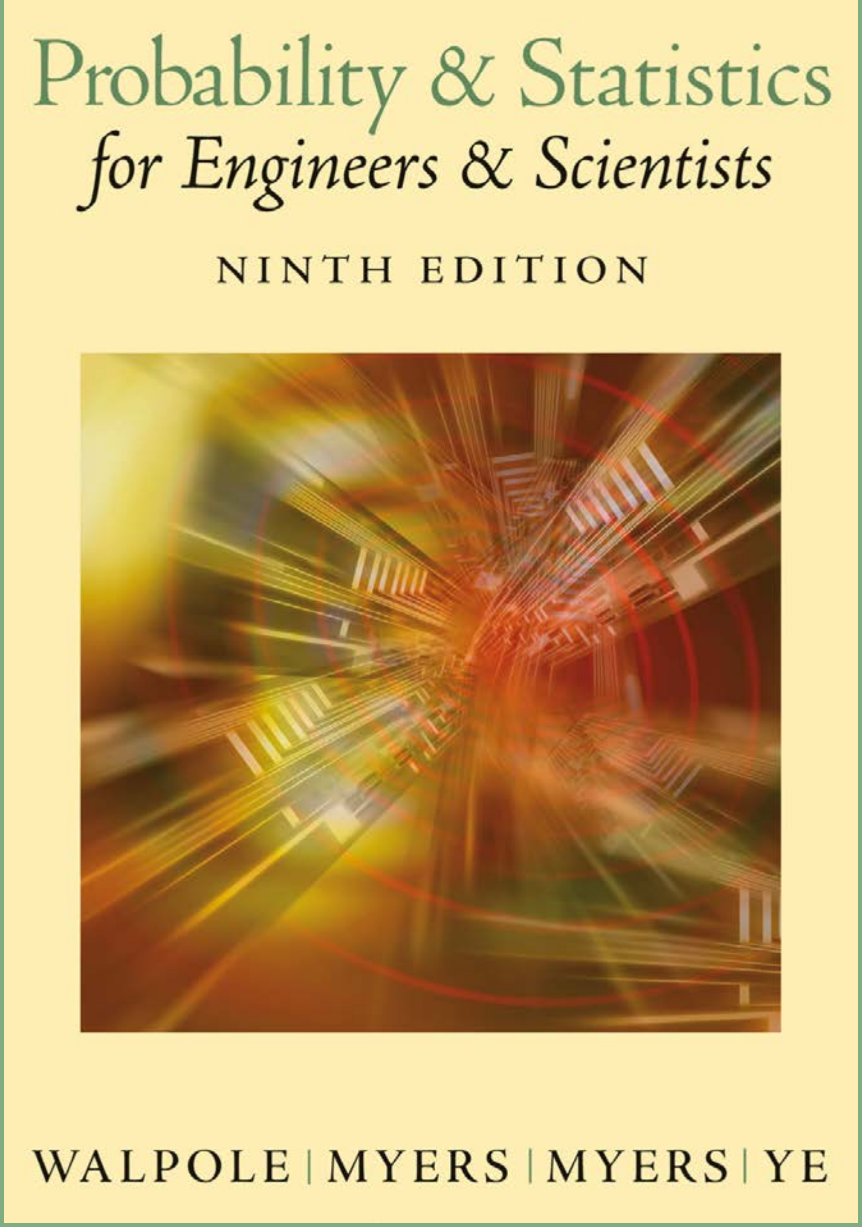


# **Figure 6.27** Area for Example 6.15



# Section 6.6

Gamma and Exponential Distributions

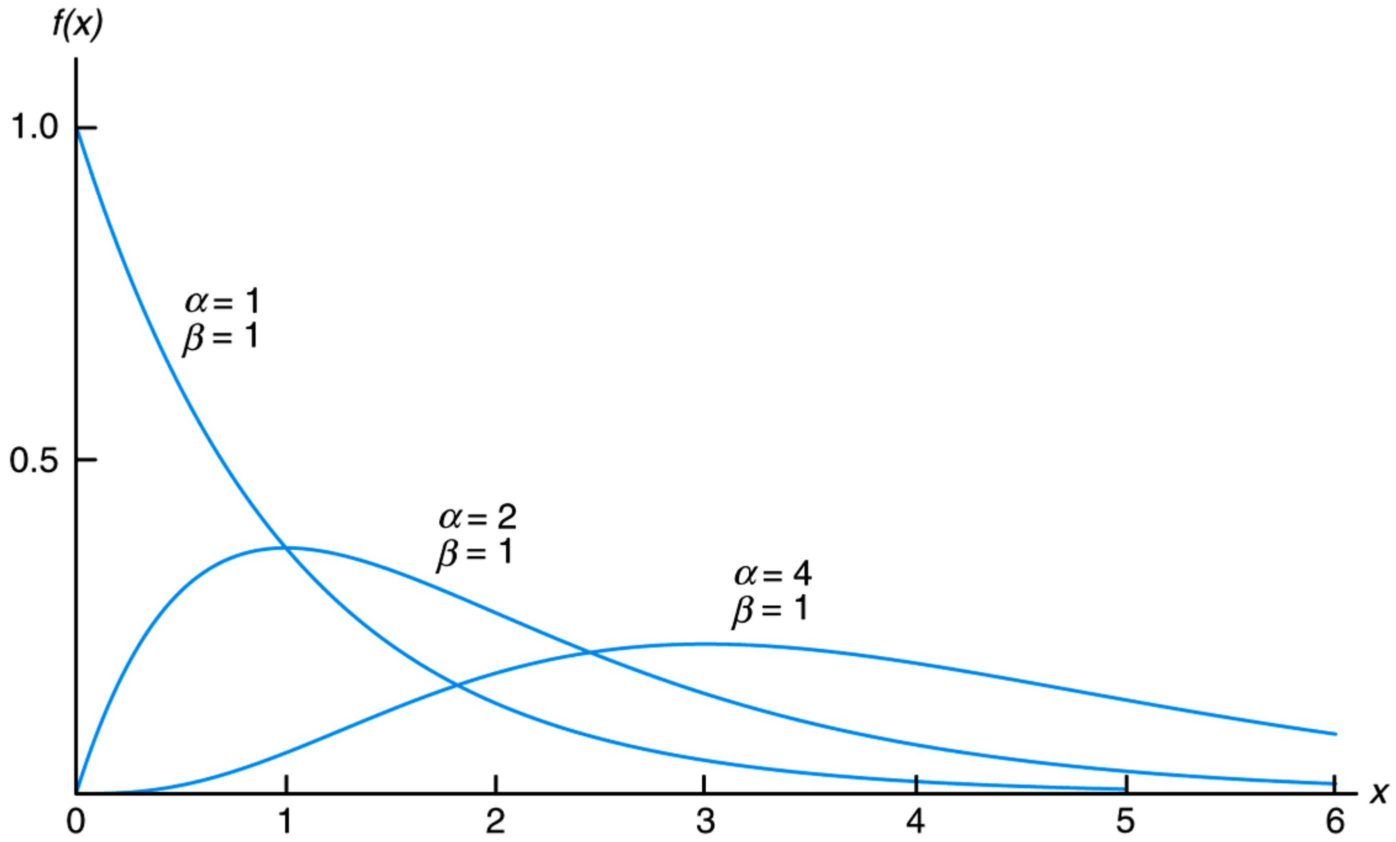


# Definition 6.2

The **gamma function** is defined by

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} \, dx, \qquad \text{for } \alpha > 0.$$

# **Figure 6.28** Gamma distributions



# Theorem 6.4

The mean and variance of the gamma distribution are

$$\mu = \alpha\beta \text{ and } \sigma^2 = \alpha\beta^2.$$

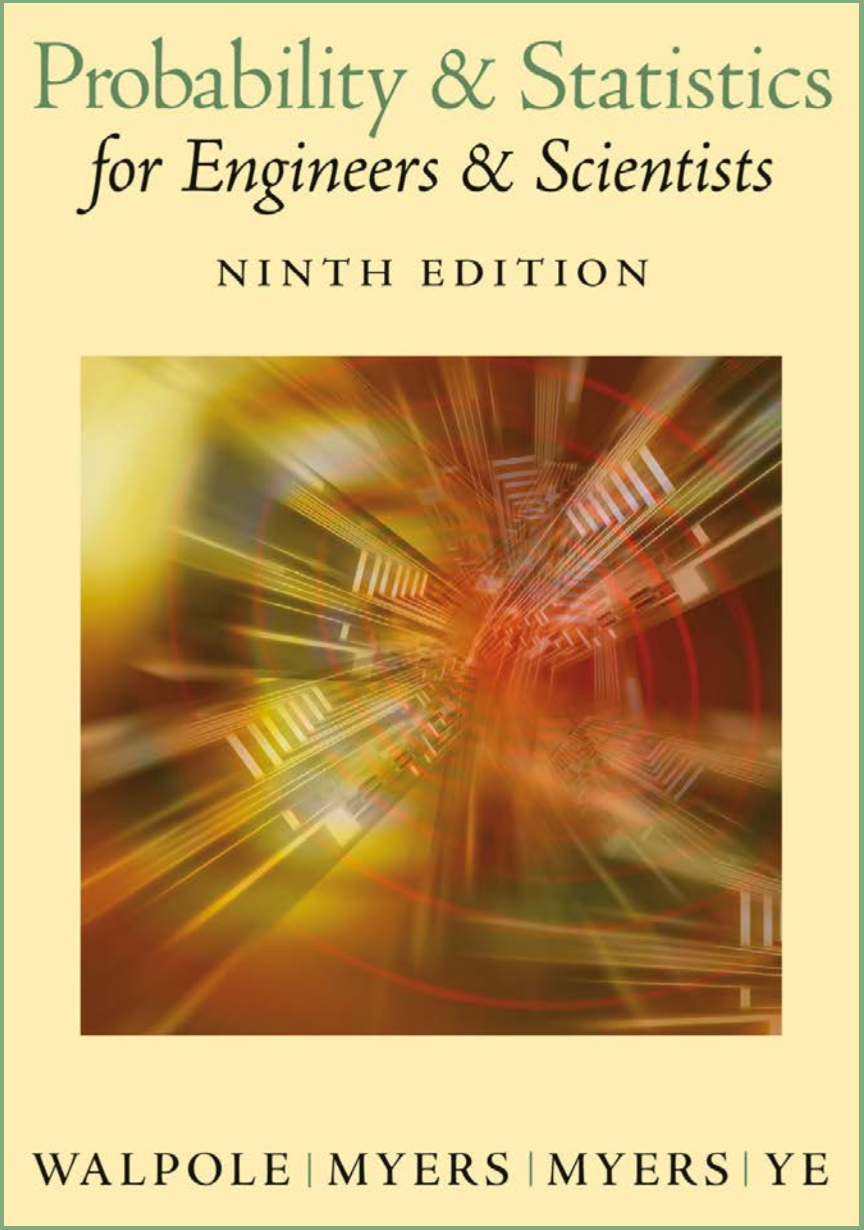
# Corollary 6.1

The mean and variance of the exponential distribution are

$$\mu = \beta \text{ and } \sigma^2 = \beta^2.$$

# Section 6.7

Chi-Squared Distributions



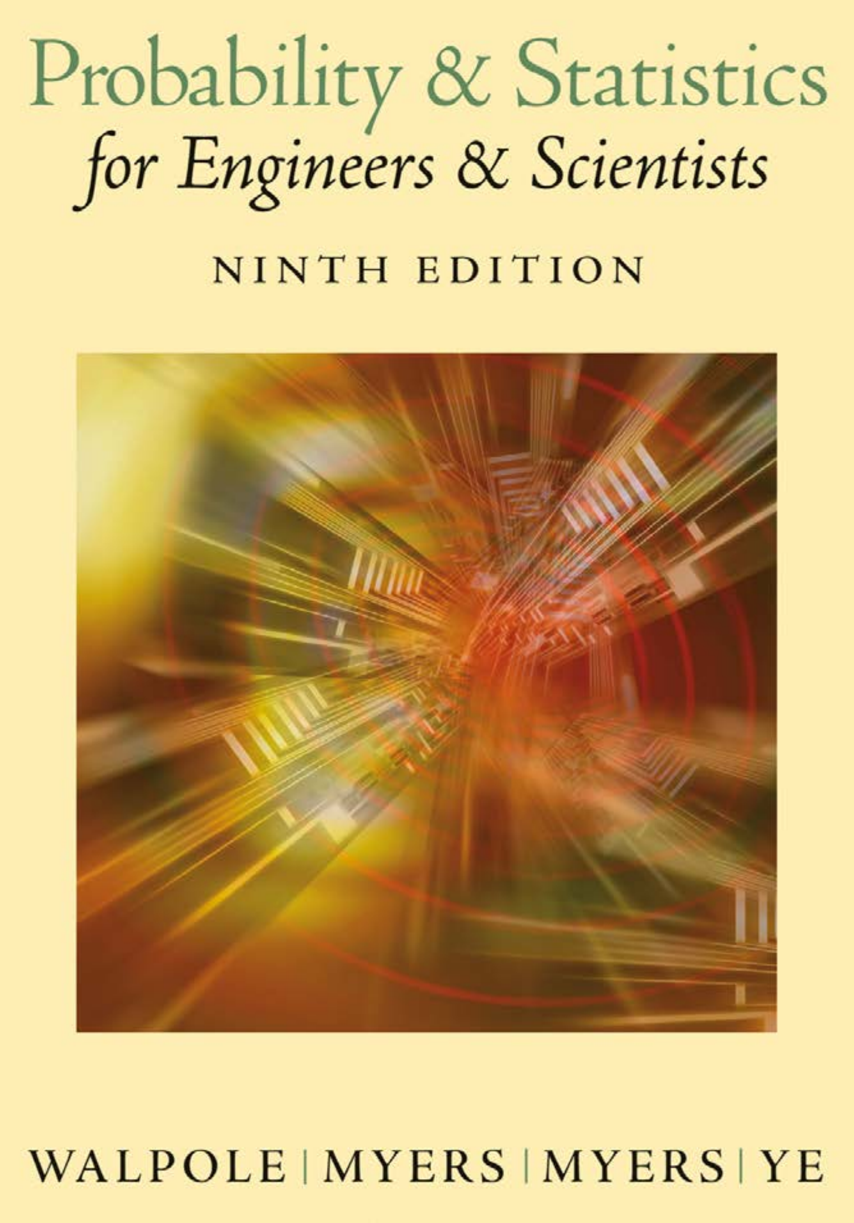
# Theorem 6.5

The mean and variance of the chi-squared distribution are

$$\mu = v \text{ and } \sigma^2 = 2v.$$

# Section 6.8

## Beta Distribution



# Definition 6.3

A **beta function** is defined by

$$B(\alpha, \beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, \text{ for } \alpha, \beta > 0,$$

where $\Gamma(\alpha)$ is the gamma function.

# Theorem 6.6

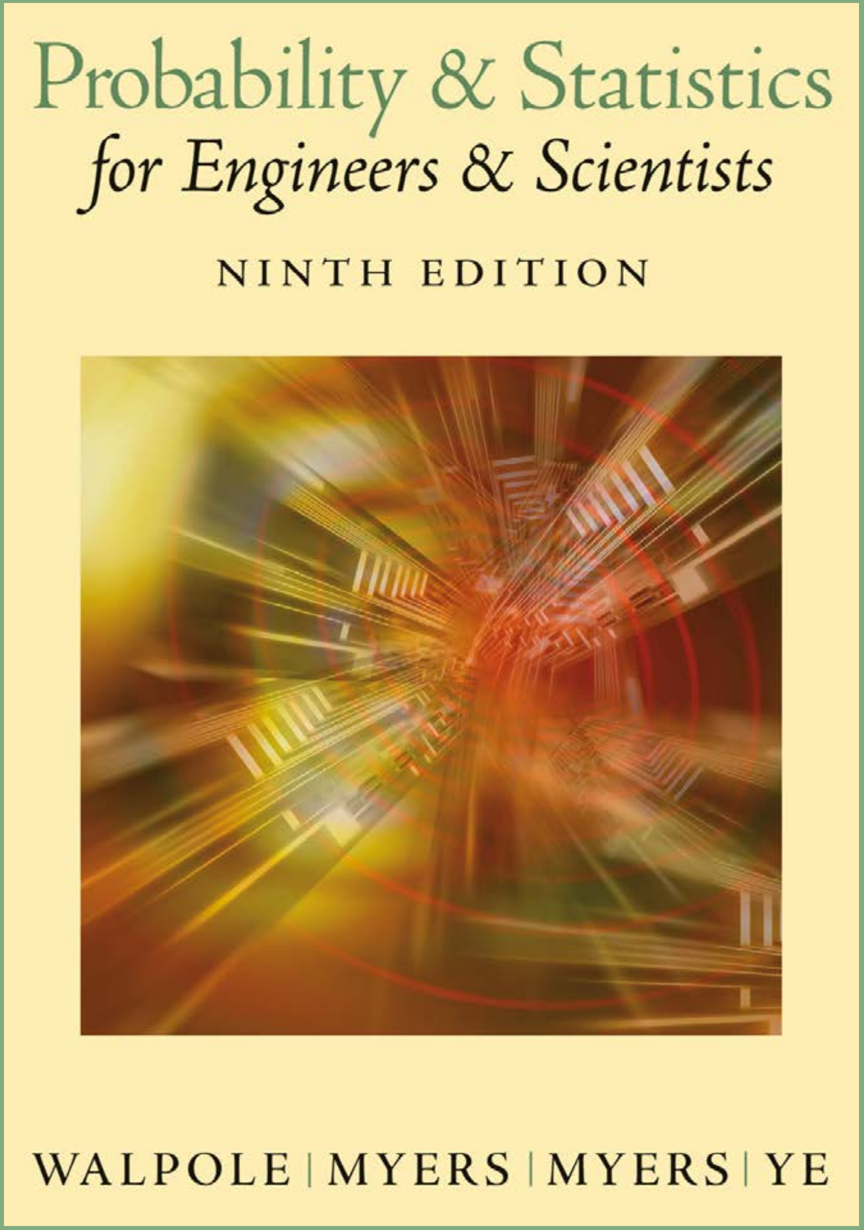
The mean and variance of a beta distribution with parameters $\alpha$ and $\beta$ are

$$\mu = \frac{\alpha}{\alpha + \beta} \text{ and } \sigma^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)},$$

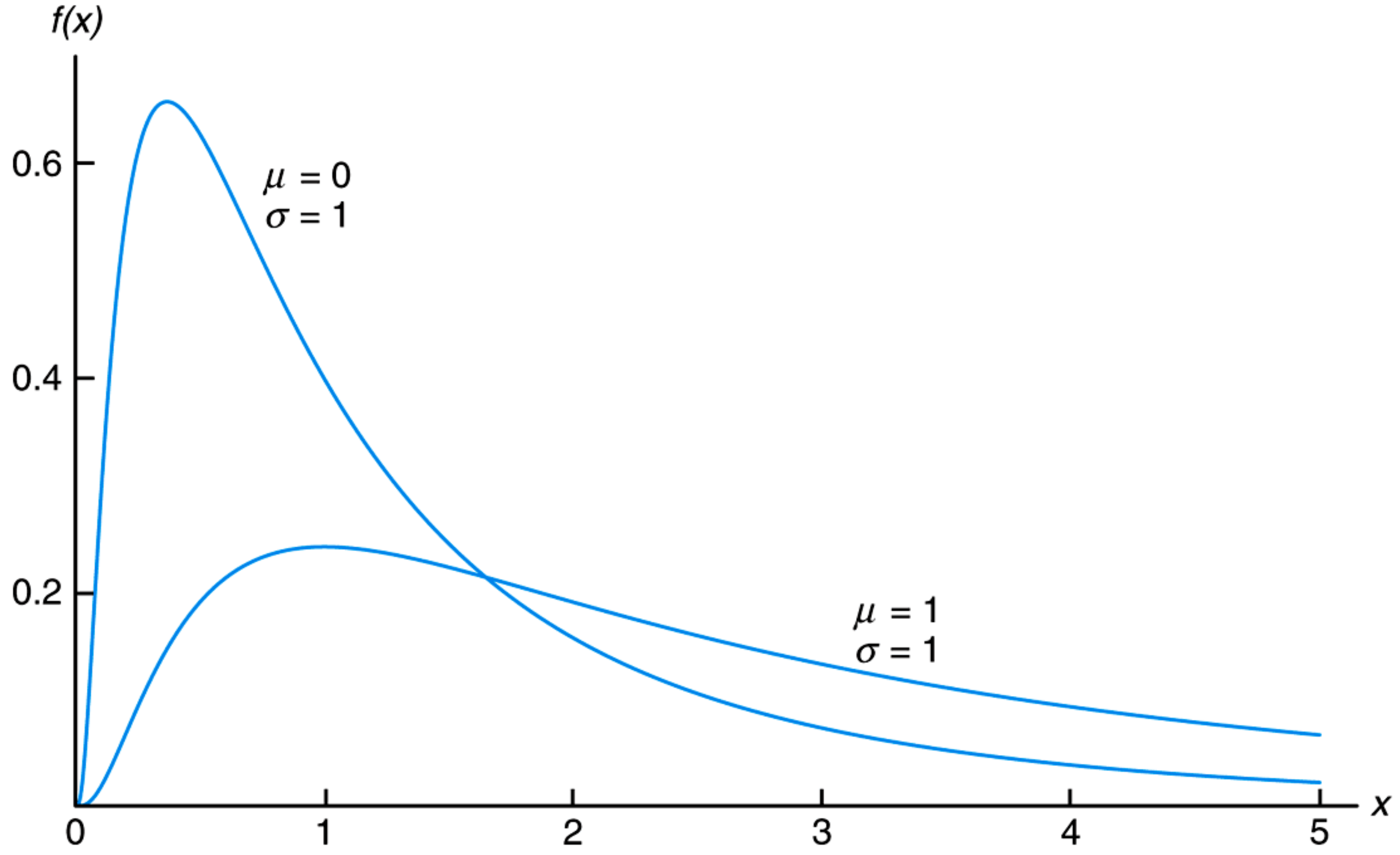
respectively.

# Section 6.9

Lognormal Distribution



# **Figure 6.29** Lognormal distributions



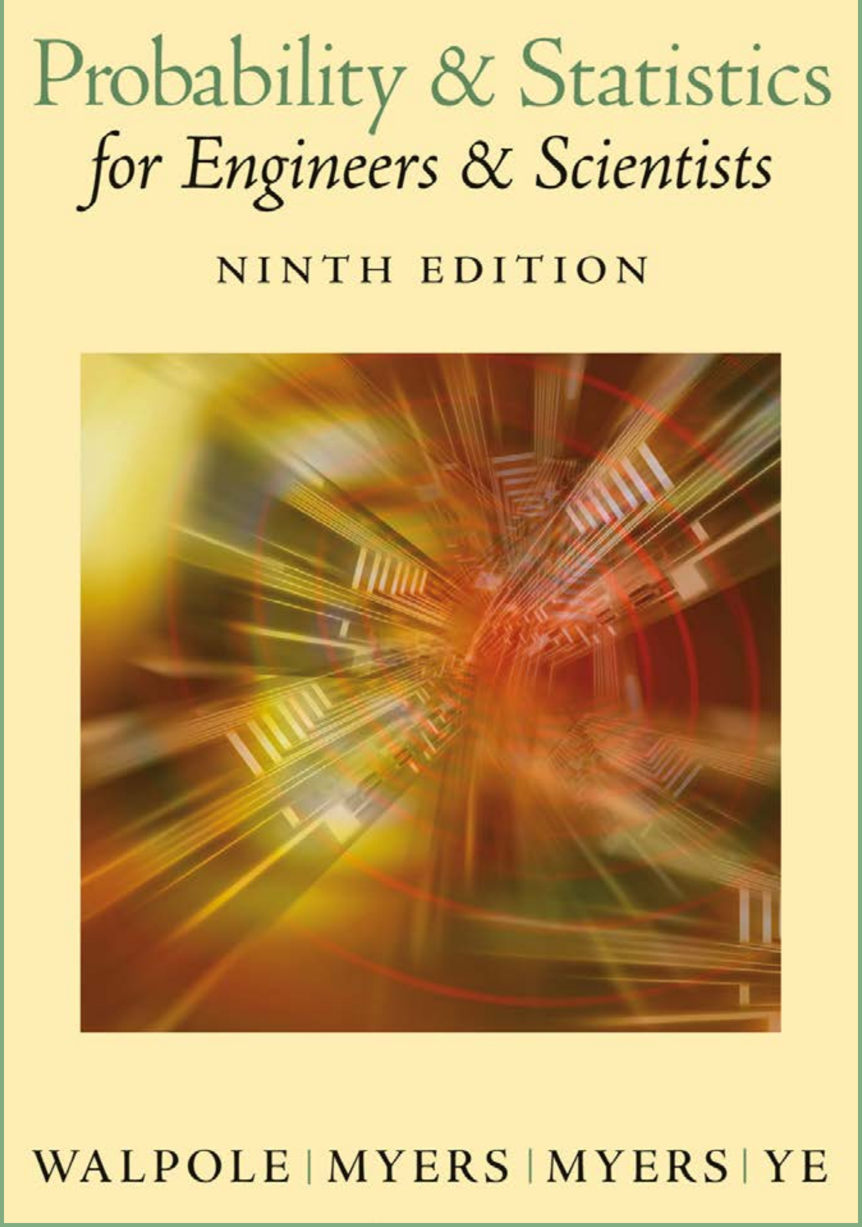
# Theorem 6.7

The mean and variance of the lognormal distribution are

$$\mu = e^{\mu+\sigma^2/2} \text{ and } \sigma^2 = e^{2\mu+\sigma^2}(e^{\sigma^2} - 1).$$

# Section 6.10

Weibull Distribution (Optional)

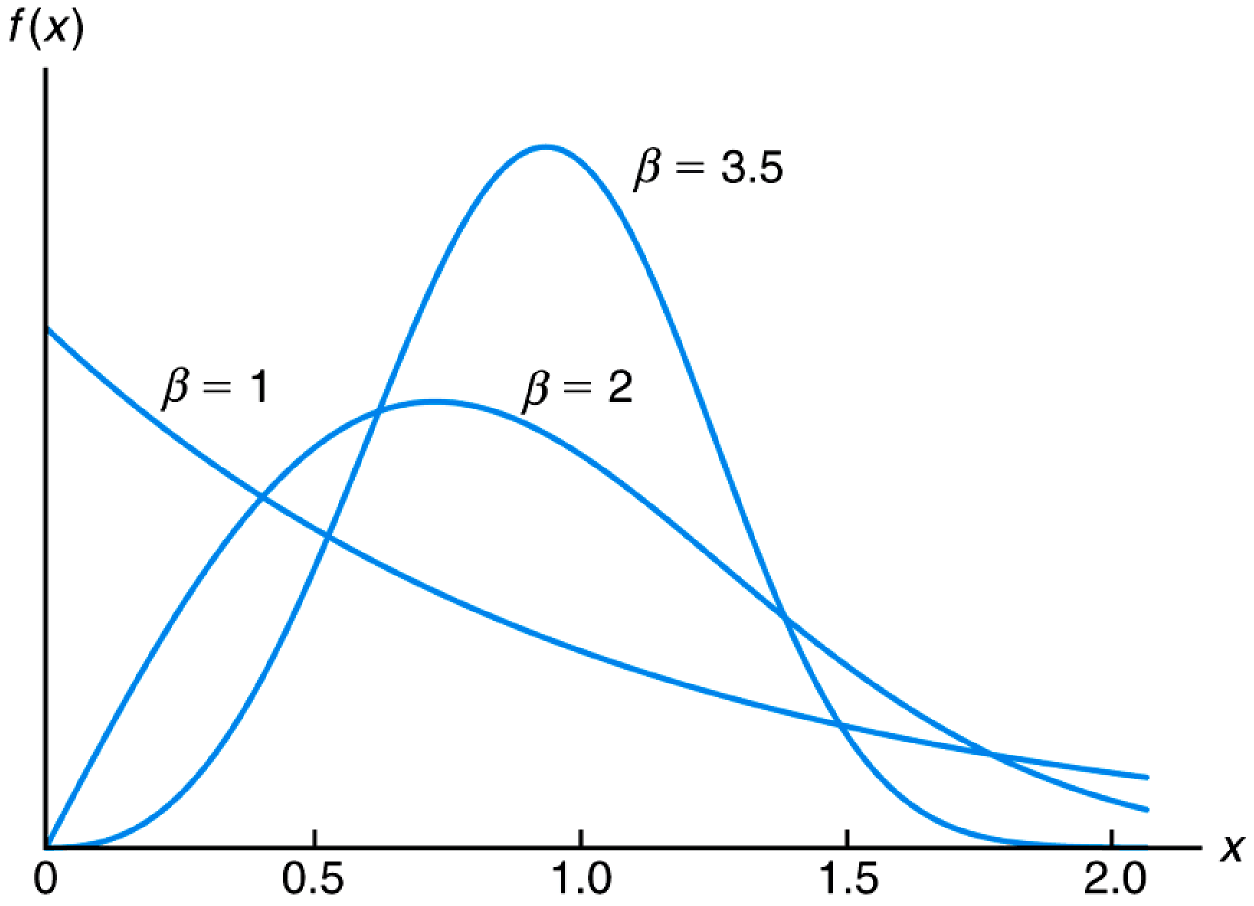


# Theorem 6.8

The mean and variance of the Weibull distribution are

$$\mu = \alpha^{-1/\beta}\Gamma\left(1 + \frac{1}{\beta}\right) \text{ and } \sigma^2 = \alpha^{-2/\beta}\left\{\Gamma\left(1 + \frac{2}{\beta}\right) - \left[\Gamma\left(1 + \frac{1}{\beta}\right)\right]^2\right\}.$$

# **Figure 6.30** Weibull distributions ($\alpha = 1$)



# Section 6.11

Potential Misconceptions and Hazards; Relationship to Material in Other Chapters